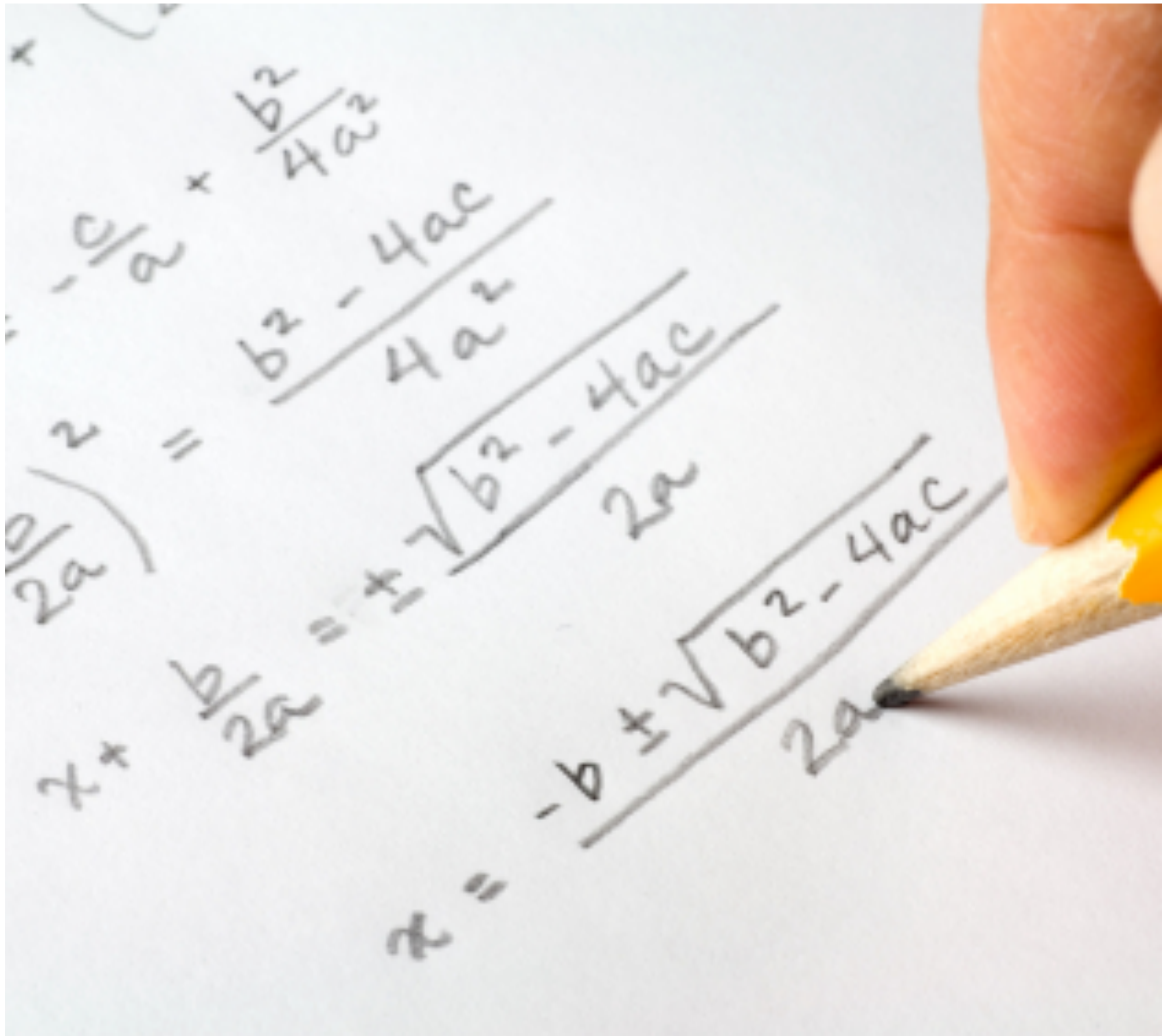
KMHS MATH DEPT.

Algebra I



MRS. ALICE ACCARDI

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SECTION 1

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1. ORDER OF OPERATIONS

Mickey Mouse: "Arithmetic is being able to count up to twenty without taking off your shoes."

Math, like so many other things in life, has rules that must be followed. Think of all the rules we follow on a day to day basis—we drive on the right side of the road, we stop at red lights and go on green, we abide by the speed limit. Well, Math is no different; there are rules we agree to obey

One of those rules is known as the Order of Operations. It's a list of the order in which we agree to perform mathematical operations when there are several in the same mathematical expression.

Here's the Order of Operations:

1. Parentheses
2. Exponents
3. Multiplication and Division, from left to right as they appear in the expression.
4. Addition and Subtraction, from left to right as they appear in the expression.

Many people refer to the Order of Operations as PEMDAS:

Parentheses

Exponents

Multiplication and Division, from left to right as they appear in the expression.

Addition and Subtraction, from left to right as they appear in the expression.

BE CAREFUL!!!! The tricky thing about PEMDAS is the last 2 steps. There are people who mistakenly think that Addition always comes before subtraction, and multiplication before division. Yet that's clearly NOT what the rules says—it says that those particular operations are performed from left to right as they appear in the expression.

EXAMPLE: EVALUATE $(6^2 + 8^2) \div 2 + 8$

$(35 + 64) \div 2 + 8$

$100 \div 2 + 8$

$50 + 8$

58

PRACTICE PROBLEMS:

1) $((11 - 2)22 \times 7) – 22$

2) $((13 - 6) +(16 \div 4)2)$

3) $14 + ((10 + 7) \times 3 2)$

4) $7 +(5 + (11 - 3)2)$

5) $((11 - 6)2 + 3) + 72$

6) $(62 + (12 \div 6 + 32))$

7) $15 + ((18 + 3) + 22)$

8) $(33 + (10 \div 5 + 22))$

9) $17 +(7 + (11 - 3)2$

10) $((11 - 2) - (15 \div 5)2)$

11) $((4 +4)^2 + 2) - 4 + 43$

12) $8 +(5 + (10 - 2)2) - 8$

13) $((6 +2)^2 \times 7) + 13 – 32$

14) $(8 \div 2)^2 + ((14 - 2) \times 42)$

15) $18 +(7 \times (9 - 3)^2) + 8$

16) $((11 + 5) +(15 \div 5)^2) \times 3^2$

17) $((17 + 7) - (8 \div 4)^2) \times 5^2$

18) $(62 + (24 \div 12 + 52)) + 2^2$

19) $(10 \div 5)^2 + ((15 + 2) + 42)$

20) $(42 + (8 \div 4 + 22)) – 4^2$

2. Basic Definitions

Charles Darwin: "A mathematician is a blind man in a dark room looking for a black cat which isn't there."

Algebra is the branch of mathematics that enables us to solve problems—specifically, to solve equations. Before we can do that, we need to define some basic terms:

- A Variable is a letter that takes the place of a number. In elementary school, you may have used symbols, say a box, instead of using variables. You've probably solved equations such as " 3 + = 7." Now, instead of that box, we'll use a letter. There are some letters that are impractical to use, simply because they look a lot like numbers. For example the letter O is probably better avoided. Likewise, a "t" can look like plus sign if you're not careful, and a lower case L looks an awful lot like the number 1.

- A Term is a mathematical expression held together my multiplication and/or division. It can contain a single number, a single variable, or a combination of numbers and variables. But as soon as you hit a "+" or a "-" sign, the term has ended.

- A Coefficient is the number in front of a variable that tells you how many you have. For example, if your expression is "2x" the coefficient is 2, and it means you have 2 x's.

- An Equation is a mathematical expression containing an equal sign. An inequality is a mathematical expression containing an inequality symbol: <, >, . , or .

- A Constant is a number that's not attached to a variable, like - 17 , 5, or 1/2 .

BE CAREFUL! Sometimes a variable will be written without an apparent coefficient—a single x or a y. Those terms DO have a coefficient; it's understood to be 1. We do the same thing in English—if I say I had a hotdog for lunch, the understanding is that it was one single hot dog. The 1 is implied.

- PEMDAS mentions exponents. As I'm sure you realize, exponents are a convenient way of showing that something has been multiplied by itself numerous times. When we use exponents, the parts of the expression all have names.

For example, if we use the expression "$2^3 = 8$", 2 is known as the BASE. It's the thing being multiplied by itself. The 3 is known as the EXPONENT… it tells us how many times the base is multiplied by itself. Finally, the 8 is known as the POWER. It tells us the result when the base is raised to the power.

3. Evaluating Expressions

Paul Harvey: "If there is a 50-50 chance that something can go wrong, then 9 times out of ten it will."

Sometimes we're asked to evaluate an algebraic expression, given particular values for the variables in a problem. For example, we may be asked to evaluate $x^2 - y$ when $x = 3$ and $y = 4$. All that means is that we substitute in the 3 for the x and the 4 for the y, and then follow PEMDAS. In this particular example, that would mean squaring the 3, then subtracting the 4. We would get : $3^2 - 4 = 9 - 4 = 5$.

EXAMPLE: EVALUATE, USING X= -6, Y = 2 AND Z =3:

$$X^2+YZ$$

$(-6)^2+(2)(3)$

$36 + (2)(3)$

$36 + 6$

42

PRACTICE PROBLEMS: EVALUATE EACH OF THE FOLLOWING, USING THE FOLLOWING VALUES:

$x = 4$, $y = 8$, $z = 2$, $w = ½$, $a = -2$, $b = -6$

1. xyz
2. $2x-3y+a$
3. $w(x+y-z)$
4. $xy+ab$
5. abw
6. $(xy)z - (x+b)^2$
7. $(ay - bz) \div (az)$
8. $zxy + bw$
9. $(bz - ay)^w$
10. $(x-y+z) \div zw$
11. $2x+3y-4z$
12. $5a+6y$
13. $w - 2(a-b)$
14. $(w+x) - (w - x)$
15. $(x+z)^2 + y^2$
16. $2w(x-5y+z)$
17. $3a-4w$
18. $a(bw+zx)$
19. abc
20. $a(b+cz)$
21. $(ab + zw)$
22. $zwy + a$
23. $bw+xy$
24. bx/y
25. $x(a-b)$

4: SETS

"Decimals have a point."

There are lots of different ways to classify numbers, and being able to classify them sometimes makes problems easier.

- A SET is a collection of things. Most of the time in math we'll be talking about different sets of numbers, but sets can contain just about anything. We can talk about the set of students in our Algebra class, the set of homerooms they belong to, or the set of towns they come from. We frequently name our sets with a single capital letter.

- The things within that set are called its ELEMENTS. The elements of a set are normally listed within parentheses and separated by commas, like this: {1, 2, 3}

- Two or more sets that contain the same number of elements are said to be EQUIVALENT SETS. So {a, b, c} and {2, 4, 6} are equivalent because each contains 3 elements.

- EQUAL SETS, on the other hand, contain the exact same elements, though not necessarily in the same order. {A, B, C} = {C, A, B} since both contain the same 3 letters.

- A set whose elements can be counted, is called FINITE. Examples of Finite sets include the set of of all Kellenberg students, the set of all US High Schools, or the set of colors that Crayola ;uses in its crayons. The number of elements doesn't have to be EASY to count, just possible to count.

- On the other hand, some sets go on for ever without ever stopping. These sets are called INFINITE sets. (That should be pretty logical; the prefix "IN" means "not".) Some examples of infinite sets are the set of all even numbers, or the set {5,10,15,20...} The "..."means the set continues on in the same pattern and never ends.

- EQUAL SETS, on the other hand, contain the exact same elements, though not necessarily in the same order. {A, B, C} = {C, A, B} since both contain the same 3 letters.

- A set whose elements can be counted, is called FINITE. Examples of Finite sets include the setthe NULL SET. We can represent it one of two ways: either empty parentheses, like this: {}, or a zero with a slash through it, like this: One example of an empty set would be the set of all KMHS students over the age of 30 (I hope!!)

BE CAREFUL!! When you refer to the empty set, be careful NOT to put the slashed zero into parentheses (like this: { }.) Since those parentheses contain something, they're no longer empty!

Sometimes we want to break a set into parts. Those parts are called SUBSETS of the orginal set. ("Sub" means "part.") A subset can contain one element or all of the elements, or anywhere in between. The symbol for subsets is a sideways "U", like this: . If we write " A ⊂ B", we're saying that set A is a subset of set B.

5. PARTICULAR SETS OF NUMBERS

Natural numbers are better for your health."

There will be times in math when you wish to refer to a particular set of numbers. Here are the sets you'll be working with this year:

- THE NATURAL (OR COUNTING) NUMBERS: { 1, 2, 3, 4,... }
This is the first set of numbers you became aware of as a child. You used them to count your fingers and toes.

- THE WHOLE NUMBERS: {0, 1, 2, 3, 4 ...}
If you had an older sibling, you probably became aware of the Whole Numbers the first time you had a toy he or she wanted: One moment you had 1 toy, and a moment later you had zero toys.

The realization that 0 was a number—and not just the absence of numbers—was actually a pretty significant development in the history of mathematics. http://www.livescience.com/27853-who-invented-zero.html

- THE RATIONAL NUMBERS: The set of Rational numbers contains any number which could be written as either a fraction or a terminating or repeating decimal. This is the set of numbers you learned as you found yourself with one cookie and one friend; you quickly and intuitively learned about the number ½.

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- THE INTEGERS: { 0, 1, 2, 3, ...}
You learned about the Integers the very first time you borrowed money, and realized that it was possible to have a bank balance of $ -1

- THE IRRATIONAL NUMBERS: As you've probably guessed, this set contains numbers which are not Rational Numbers. They can't be written as fractions, or as terminating or repeating decimals, even if you can figure out the pattern behind the digits. Examples of irrational numbers are 0.112123123412345... and 1.1113151719...

- THE REAL NUMBERS: The set of Real Numbers contains all the numbers you've read about up until this point: It includes Natural Numbers, Whole Numbers, Integers, Rational Numbers and Irrational Numbers.

- THE IMAGINARY NUMBERS: OK, technically this one is cheating. You won't really be learning about the Imaginary numbers until you take Algebra II and Trig. So I'm not going to tell you what it includes, though a quick check of Google should provide you with an easy explanation. Are you curious enough to check??

6. OPERATIONS WITH REAL NUMBERS

"To a mathematician, real life is a special case.

In elementary school, you learned the rules for adding, subtracting, multiplying and dividing positive numbers. The rules become a little more… let's say "interesting" when we start to involve negative numbers. Here are the rules for working with numbers of varied signs:

- MULTIPLICATION: The product of two positive numbers is positive, as is the product of two negative numbers. If you multiply two numbers of different signs, however, the product is negative.

- DIVISION: Division works just like multiplication: The quotient of two positive numbers, or of two negative numbers, is positive. If you divide two numbers of opposite signs, the quotient is negative.

- ADDITION: Addition is where it starts to get a bit confusing. If you're adding two numbers with the same sign, take the sign and just add the numbers. (So -4 + -3 = -7 for example.) If you're adding two numbers of opposite signs, you take the sign of the larger, and then subtract the numbers. (So -2 + 5 = +(5-2) or 3. -6 + 4 = -(6-4) or -2.)

- SUBTRACTION: Subtraction is merely adding the opposite. So your first step is to rewrite (or, at least, rethink) the problem as the first number plus the opposite of the second. Then follow the rules for addition.

BE CAREFUL!!! You will NEED to be able to work quickly and accurately with numbers of various signs throughout the rest of your high school career. If you're not already good with negative numbers, you NEED to put in the time and effort to become comfortable and accurate with them. Trust me, you'll be very glad you did. (Oh, and while we're on the topic, I hope you KNOW your times tables. If not, then it's time to break out the flashcards. Please believe me on this subject!!!!!)

PRACTICE PROBLEMS:

1. -16 + 34
2. -44+ (- 12)
3. 12 +(-3)
4. -3 + -6
5. 15+ (-4)
6. -3 + (-5)
7. -11 + 23
8. 23 + (-3)
9. -10 + 10
10. 4 + (-3)
11. 12 – (-3)
12. 1- 5
13. -7 -7
14. -23 – 12
15. 22- (-4)
16. _45 – (-3)
17. -22 – 17
18. 6 – (-6)
19. 9 – 14
20. 11 – (-11)
21. 6(-7)
22. _ 8 (-9)
23. 24(-2)
24. (-50)(-2)
25. (30)(-3)
26. (-17)(6)
27. (-131)(-2)
28. (16)(-3)
29. (28) (-5)
30. (90)(-2)
31. (-280)/4
32. 320/ (-8)
33. (-569)/70
34. (-369)/(-18)
35. 72/(-12)
36. (-169)/13
37. (-48)/6
38. (-63)/(-9)
39. (-39)/(-13)
40. (-125)/(-5)

41. -15 +5

42. 32/ (-8)

43. -23 - (-78)

44. 64 / (-8)

45. -45 / (-9)

46. (-82) - (-19)

47. -26 + (-4)

48. -32 (-2)

49. 43 + (-72)

50. -10 / (-5)

51. -16/2

52. -21 -6

53. 22- 48

54. -121 / (11)

55. -144 /(-4)

56. 66/(-22)

57. 123 - (-24)

58. -134 +56

59. -78 - 32

60. -14 + -56

61. -13 (-13)

62. (-12)(-3)

63. -90 - 30

64. -12 + 43

65. -28 - (-20)

66. -26 + 26

67. -56 -56

68. 100/ (-25)

69. (-16)(16)

70. -79 +38

71. 43 - (-24)

72. -144 + 58

73. -124 / (-4)

74. 128/4

75. -300 /(-25)

7. PROPERTIES OF REAL NUMBERS

"Mathematics is like love, a simple idea, but it can get complicated."

There are certain rules that numbers always follow, known as Properties. Here are the properties of real numbers:

- REFLEXIVE PROPERTY. Any number equals itself. 3=3, -10 = -10, x=x. It's going to be particularly important next year in Geometry.

- SYMMETRIC PROPERTY: An equation can be reversed. If you solve an equation and get 4=x. then you're correct in telling me that x=4.

- TRANSITIVE PROPERTY. If two things are equal to the same thing, then they're equal to each other. So if 100/50 = 20/10, and 20/10 = 2/1, then 100/50 = 2/1.

- COMMUTATIVE PROPERTY OF ADDITION: You can reverse the order when you're adding numbers. So 2+9 = 9 + 2. (If you know anyone who COMMUTES to New York, they travel from your neighborhood to NYC, and then back again. Commuting means going back and forth.)

- COMMUTATIVE PROPERTY OF MULTIPLICATION: Just as with addition, the order of numbers can be changed when you're multiplying. Remember when you were memorizing your times tables, and realized that you already knew a lot of the big ones? That was because of the Commutative Property of Multiplication. If you had already memorized 2 * 8, you didn't have to go back and memorize 8*2; the answers were the same.

- - ASSOCIATIVE PROPERTY OF ADDITION. Your associates are your group. The Associative Property of Addition says you can move the grouping symbols in an addition problem. So (2+3)+4 + 2+ (3+4).

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- ASSOCIATIVE PROPERTY OF MULTIPLICATION: As you've probably figured out, this means you can change the grouping symbols in a multiplication problem without changing its answer. So 2(5 *8) = (2*5) *8.

- ADDITIVE IDENTIY: The additive identity says you can add 0 to any number without changing the value of the number. 0 is known as the Additive Identity Element.

- MULTIPLICATIVE IDENTIY: The multiplicative Identity (ask your teacher how to say that!) means you can multiply any number by 1 without changing the value of the number. The Multiplicative Identity Element is 1.

- ADDITIVE IDENTIY: The additive identity says you can add 0 to any number without changing the value of the number. 0 is known as the Additive Identity Element.

- MULTIPLICATIVE IDENTITY: The multiplicative Identity (ask your teacher how to say that!) means you can multiply any number by 1 without changing the value of the number. The Multiplicative Identity Element is 1.

- ADDITIVE INVERSES: Additive Inverses are opposites, two numbers whose sum is zero.

- MULTIPLICATIVE INVERSES: Multiplicative inverses are reciprocals, two numbers whose product is 1.

- DISTRIBUTIVE PROPERTY: The distributive property says that when you multiply numbers inside parentheses by an outside number, EACH number inside must be multiplied. For example,
3(x+2) = 3x + 6.

BE CAREFUL!!! If you choose to abbreviate the properties, be particularly careful with the Associative Property. Go to at least the "C".

PRACTICE PROBLEMS: Identify the property shown in each example.

1. $3+6= 6+3$
2. $2=2$
3. $a(bc) = (ab)c$
4. $2(\frac{1}{2}) = 1$
5. If $a=b$ and $b=c$, then $a=c$
6. $2(x+y) = 2x+2y$
7. If $3=x$, then $x=3$
8. $7(8) = 8(7)$
9. $17+0 = 17$
10. $(2+7) + 3 = 2+ (7+3)$
11. $2(1)=2$
12. $-6 + 6 = 0$

In the following problems, identify the correct applications of the Distributive Property. For those that are incorrectly applied, please make the proper correction.

13. $a(x + y) =ax + y$
14. $(2 + a)b= 2b + ab$
15. $6(x - y) =6x - y$
16. $x(2+y) = x^2 + xy$
17. $7(a+ b + c) = 7a+7b+7c$
17. $30x^2-40x^3+16x^2-22x^3$
18. $-x +7y +2x -9y$
19. $24xy + 7x^2-4xy +3x^2$
20. $3x + 4y -5x -8y +12x -18y$

Give an example of why each of the following operations does NOT have the property listed.

21. Subtraction, commutative
22. Division, commutative
23. Subtraction, associative
24. Division, associative

8. LIKE TERMS

'FOUR' is the only number in the English language that is spelt with the same number of letters as the number itself.

One of the most basic ideas in Algebra is the idea of Like Terms. Like terms have :

- The same variables
- Raised to the same powers

Their coefficients do NOT have to match. So, for example, $3x^3$ and $-6x^3$ are like terms (since they contain the same variable raised to the same power) but 5y and 5x are not (since they don't have the same variable raised to the same powers.)

The rule in Algebra is that you can ONLY add or subtract like terms!!! The way to do that is to combine the coefficients. When you're adding or subtracting, you don't change any exponents.

Example 1: Simplify: $6x^2 + 8x^2 - 2x^2$

Answer: Combine the coefficients: 6+8 -2= 12. The answer is $12x^2$

Example 2: Simplify: $5x + 7y - 2x - y$:

Answer: Work first with one variable and then with the other. When I combine the x's, I get 3x. When I combine the y's, I get 6y. So the answer is 3x + 6y. Those two terms can't be combined since they're not like terms.

PRACTICE PROBLEMS: SIMPLIFY:

1. $4x + 3x + 6x$
2. $2x+6-5x+3$
3. $12x+4x-21x+16x$
4. $2x-y+5x-y$
5. $3x+5y+2x-y-5x-4y$
6. $3x-y-x+y$
7. $22x+72y+10x-23y$
8. $x^2+6x-4x^2+2x$
9. $5x^3-2x^2+8x+7x^3-8x-x^2$
10. $3xy^2+3x^2y+13xy^2-4x^2y$
11. $25x^2y^4-21x^2y^4$
12. $22xyz + yz-3yz+ 12xyz -4xy$
13. $3a+4b -3a + 3b + a -b$
14. $12x - x^2+3x+8x^2$
15. $23x-6x^2+7x-18x^2$
16. $13x+7y -2z +8z -6y+12x$
17. $30x^2-40x^3+16x^2-22x^3$
18. $-x +7y +2x -9y$
19. $24xy + 7x^2-4xy +3x^2$
20. $3x + 4y -5x -8y +12x -18y$
21. $3x+5y -4x +7y -x+12y$
22. $14x - 80y -35x + 55y$
23. $x+8y-3+4y -6 +11x -y +10$
24. $2x+y +7y +x -5y -x +12y +x$

9. ONE STEP EQUATIONS

What do mathematicians eat on Halloween?

Pumpkin Pi.

To "solve an equation" means to find an answer for the variable.

For these early equations, what we want to do is to get the variable on one side of the equation all by itself. We do that through the use of "Inverse Operations." In simple language, that means doing the opposite. (Hey, you're teenagers. You should be used to doing the opposite of what you're told to do J) If the x is being multiplied by 2, you divide both sides of the equation by 2 to cancel out the coefficient. If the x is being added to 6, you subtract the 6 from both sides.

BE CAREFUL!!!! When we say "both sides of an equation" we mean both sides of the equal sign. The equal sign splits the equation into a left and right side. **ANYTHING you do to one side must always also be done to the other side!!!**

Example 1: **$4x = 20$** Since x is being multiplied by 4, we need to do the opposite—divide both sides of the equation by 4. That gives us = , or x=5

Example 2: $3 + y = 8$. Since y is being added to 3, we need to subtract 3 from both sides. That gives us $3+y - 3 = 8 - 3$. On the left side, the 3's cancel each other out. We get $y = 5$.

BE CAREFUL!!! Sometimes the coefficient is a fraction. According to the rules, if you want to eliminate a coefficient, you divide both sides by that coefficient. But think back to your elementary school math: dividing by a fraction is the same thing as multiplying by its reciprocal. You'll find it MUCH easier if you get into the habit of eliminating fractional coefficients by multiplying both sides by their reciprocal.

Example 3: **=18**. Instead of dividing both sides by 2/3, let's multiply both sides by its reciprocal, 3/2. (= 18( ) , or x = 27

PRACTICE PROBLEMS: SOLVE EACH EQUATION USING INVERSE OPERATIONS:

1. $3y = 27$
2. $x + 5 = 32$
3. $x - 7 = 12$
4. $6x=72$
5. ½ $x = 16$
6. $7y=28$
7. $y-12 =34$
8. $x+6=6$
9. $8y=48$
10. $-2x=12$
11. $Y+34 = 56$
12. $W- 45 = 74$
13. $16 - x = 5$
14. $94 - x = 100$
15. $15x = 90$
16. $X+ 4.5 = 7.8$
17. $7x=-56$
18. $2x/3 = 12$
19. $-x/2 = 42$
20. $5x = -200$
21. $x+3 = 27$
22. $16x = 48$
23. $-6x = 72$
24. $13 +x = 29$
25. $10 + x = 93$
26. $18 = 16 + x$
27. $-19 = -12 +x$
28. $100 = -25x$
29. $12x = -144$
30. $150 = 50x$
31. $100 = -96 + x$
32. $84 = 7x$
33. $56 = -8x$
34. $16 = 20 +x$
35. $55 = 11x$

10. CHECKING YOUR ANSWER

Why did the math book look so sad? Because it had so many problems.

To check the answer to an equation, plug your answer back into the ORIGINAL equation. Do Order of Operations on each side of the equation. At the end of the process, you should have the same number on the left side as you do on the right.

BE CAREFUL!!! When you check an equation, you'll end up with some number equal to itself That number is NOT the answer to the problem!!! The answer is the number you substituted in!!!

PRACTICE PROBLEMS: SOLVE AND CHECK EACH EQUATION:

1. 16 + x = 92
2. 43 – x = -12
3. 7x = 84
4. -3x = -45
5. 3-x=3
6. 17+x = 15
7. -4x = 68
8. 15+x = 17.5
9. 83 – x = 39
10. 16 + x = -37
11. 39 – x = -42
12. -26x = 1-4
13. 15x = 225
14. 482 + x = 374
15. 25x = - 875
16. 3x = 480
17. 255 = -5x
18. 1040 = -4x
19. 9x = 819
20. 22x =264
21. 15+x = 22
22. 12 + x = -5
23. 5x = -75
24. 4x= 0
25. 12 + x = 20
26. -3x = 21
27. 12 + x = 21
28. 14x = -28
29. 17 + x = -17
30. 30x = 15

11. SOLVING TWO-STEP EQUATIONS

Why did the boy eat his math homework?

Because the teacher told him it was a piece of cake

Some equations are a little more complicated than the one-step ones you've seen so far. That doesn't necessarily make them more difficult, merely longer. The equations we're dealing with today contain both a variable, usually with a coefficient, as well as a constant on the same side.

In order to solve a two-step equation, first use inverse operations to remove the constant. Then use inverse operations again to remove the constant.

EXAMPLE 1: 4X + 6 = 18.

ANSWER: First, let's deal with the constant, 6. It's being added, so we do the opposite and subtract it from both sides: 4x + 6 – 6 = 18-6. That gives us 4x = 12. Now we use inverse operations again: we divide both sides by 4. That leaves us with x = 3.

PRACTICE PROBLEMS: SOLVE AND CHECK:

1. 3x + 6 =81
2. 5x-2 = 78
3. 6+6x=240
4. ½ x + 5 = 36
5. 16 – 2x = 20
6. 9x+3 = 66
7. 4x+ 120= 148
8. 5x-12 = 48
9. 7-2x = -9
10. 15 – 3x = 45
11. 8x+ 4 = -28
12. 2x/3 + 6 = 34
13. 14 + 3x = 32
15. 16 = 4x – 20
16. 24x + 4 =220
17. 9x – 3 = 60
18. 3x + 5 = 29
19. 66 = 11x+11
20. 4x + 8 = 324
21. 3x+4 =25
22. 5x-9 = 71
23. -3x+2 = 20
24. 6-2x = -50
25. -22 = 2-4x
26. 3x+12 = 72
27. -15 = 6-3x
28. 5-6x = 35
29. -16 =32 -4x
30. 8-2x = 14
31. 3x+20 =86
32. 5x+25 = -75
33. 16-2x = 24
34. 12x -36 = 60
35. 4x - 28 = 16
36. 16x -48 = 80
37. 12x -4 = 8x
38. 5x+15 = 80
39. 20x -100 = 400

12. PRELIMINARY VERBAL PROBLEMS

Have you heard the latest statistics joke?

Probably.

Once you graduate from school, it will probably be pretty rare that you actually come across an equation. That doesn't mean you won't do math, merely that it will be math in the disguise of a real life situation. Whether you're planning a vacation or buying a car, you ‘ll need the ability to translate a real life situation into an algebraic equation. We call these real life situations Verbal Problems.

Every verbal problem begins with a "LET" statement. That's where you tell the reader (and remind yourself) what the variable represents. In the beginning, it will almost always be the thing you're trying to find, but that won't always be the case.

Next, you need to translate the words in the problem into mathematical operations. For example, the phrase "The sum" will mean addition, while "The difference" will mean subtraction.

I could just give you a list, but let's do it this way instead: Make 5 columns, as shown below. And in each, list the phrases that might be used to indicate that particular operation. I've gotten you started, but there are many more than the three I've listed.

+	-	X	/	=
sum				
Add ed to				
Incr eas ed by				

Remember, too, that Verbal problems should ALWAYS be answered verbally—that means a grammatical English sentence.

PRACTICE PROBLEMS. SET UP A LET STATEMENT AND AN EQUATION FOR EACH PROBLEM. THEN SOLVE EACH EQUATION AND EXPRESS THE ANSWER IN A SENTENCE:

1. Three more than a number is 8. Find the number.
2. The product of 8 and a number is 72. Find the number.
3. Six less than a number is 15. Find the number.
4. A number, increased by 26, is 57. Find the number
5. A number, lessened by 25, is -36. Find the number.
6. Twenty eight is the product of 2 and a number. Find the number.
7. Kerri's age, decreased by 2, is 26. How old is Kerri?
8. Five less than the product of 4 and a number is 15. Find the number.
9. The product of -2 and a number, increased by 4, is 30. Find the number.
10. Half a number, increased by 8, is 13. Find the number.
11. Six times a number, decreased by 12, is 42. Find the number.
12. As of February 14, 2014, Levittown had received 54 inches of snow for the season. That amount was 4 inches more than twice the normal amount. What's the normal annual snowfall for Levittown?
13. The height of the Sears Tower in Chicago is 201 feet more than the height of the Empire State Building. If the height of the Sears Tower is 1451 feet, how tall is the Empire State Building?
14. According to visitorlando.com, in 2009 Orlando had a total of 46,583,000 visitors. In 2011 (the last year of their statistics), there were 55,168,000 visitors. How many more people visited Orlando in 2011 than in 2009?
15. Mike is 3 more than six times as old as his cousin Tommy. If Mike is 21 years old, how old is Tommy?
16. Over the course of his career, Lou Gehrig had a total of 1995 RBIs. That number, 218, is the number of career RBIs by Babe Ruth. How many career RBIs did Ruth have?
17. According to https://www.ohe.state.mn.us/dPg.cfm?pageID=948 , the median annual earnings by someone with a Bachelor's Degree is $50,433. That amount is $8833 less than twice the earnings of someone with only a high school diploma. How much can someone with only a high school diploma expect to earn?
18. The price of a postage stamp in 2014 was 4 cents more than 9 times the price in 1963. If the 2014 price ws $0.49, how much was a postage stamp in 1963?

13. EQUATIONS WITH PARENTHESES

Blaise Pascal: "It is not certain that everything is uncertain."

Some equations contain an expression in parentheses being multiplied by some number. (As in "twice the sum of a number and 10…) When that happens, we need to use the Distributive Property. As you will remember, it states that when a number or variable is multiplying parentheses, each term inside the parentheses gets multiplied by that number or variable. So we distribute, then use inverse operations to solve the equation.

BE CAREFUL!!!! Sometimes you'll see a negative sign outside parentheses. What that means is that you're to distribute a -1. Write in the 1 between the negative sign and the parentheses, and distribute that -1. Don't forget!!!

PRACTICE PROBLEMS: SOLVE AND CHECK EACH EQUATION.

1. $2(x+4) = 10$
2. $3(2x+7) = 81$
3. $5(3x-3) = 90$
4. $12(5-x) = 72$
5. $7(x+3) = 56$
6. $-4(x+3) = 20$
7. $-3(2x+1) = 39$
8. $5x-2(x+1) = 13$
9. $10x-4(x+6) = 84$
10. $2x - (3x+12) = 23$
11. $14 - (x+3) = 15$
12. $12x - (3x+9) = 99$
13. $22x - (4x+9) = 63$
14. $5x - (4x+13) = 46$
15. $7x - 2(2x+12) = 120$
16. $-6x+4(3x-36) = 240$
17. $14x - 3(5x-7) = 84$
18. $9x - (2x+7) = 70$
19. $8x - (2x+12) = 126$
20. $5x - (3x-24) = 62$
21. $3(x-8) = 90$
22. $10 - 5(2x+1) = 75$
23. $12+3(5x-2) = 96$
24. $6-(3x+1) = -3$
25. $7-(x+1) = 7$
26. $29 - (2x+5) = 16$
27. $23 = 4(x+1)+3$
28. $28 = 3(2x+1) + 1$
29. $35 = 5(2x+3) + 10$
30. $-15 = 3(x+4) - 12$
31. $4x-3(-5x-9) = -182$
32. $2x+7(-x-6) = -92$
33. $-5x+3(3x-7) = -45$
34. $-4x-3(-x+16) = -8$
35. $4x+7(4x+19) = -59$

14. VARIABLES ON BOTH SIDES OF AN EQUATION

How did the soccer fan know before the game that the score would be 0-0?

The score is always 0-0 before the game.

Some equations contain variables on both sides of the equal sign, and frequently they have a constant on each side as well. Your objective is still the same: to isolate the variable on one side of that equal sign.

Honestly, it doesn't matter which term you move first. But for the sake of giving you a system, let's go with this process:

- First, move the smaller amount of the variable—use inverse operations to get all the variables on one side of the equation.

- Next, move the constants to the OTHER side of the equation, using inverse operations again.

- Finally, use inverse operations to remove the coefficient.

BE CAREFUL!!! If you happen to subtract everything from one side of the equation, don't panic! All that means is that there's a zero on that side. What you'll want to do next is to use inverse operations to move any term from the other side to the side with the zero.

EXAMPLE: $4x + 12 = 6x + 20$

$4x + 12 - 4x = 6x + 20 - 4x$

$12 = 2x + 20$

$12 - 20 = 2x + 20 - 20$

$-8 = 2x$

$-8/2 = 2x/2$

$-4 = x$

FYI…On your computer there are very likely files with the extension .gif or .jpg or .zip. These are files that are compressed to reduce the size of the files. The particular extension identifies the algebraic algorithm that is used to compress the file and the algorithm that is needed to uncompress it. How can so much information be stored on your iPod? This is again compressed information, compressed using an algebraic expression.

PRACTICE PROBLEMS: SOLVE AND CHECK EACH EQUATION:

1. $3x+5 = 4x$
2. $2x - 6 = 8x$
3. $9x - 6=12x$
4. $5x - 15 = 35x$
5. $12x - 80 = 2x$
6. $3x+5 =4x+2$
7. $16x + 4 = 20x - 16$
8. $13x + 22 = 14x + 2$
9. $5x+15 = 15x -35$
10. $3x + 18 = 9x - 54$
11. $2(x+ 1) = 3(x-4)$
12. $5(2x-4) = 2(3x + 12)$
13. $2x + 3(x+1) = 6x$
14. $4(2x+12) - 2x = 4(x-3)$
15. $9(2x+4) = 6(x-1)$
16. $2(3x-4) = 6x+5$
17. OK, number 16 was a trick question. Explain what happened and why. What does that mean about the solution to the equation?
18. $4(2x+3) =6(x+8)$
19. $8(x+3) + 2x = 12x -6$
20. $3(x+1) -2(x-1) = 2x$
21. $3x+12=5x-18$
22. $12x-4=4x+20$
23. $16x-20=18x+4$
24. $25x-75=20x+15$
25. $4-3x=16+x$
26. $6(2x+2)=4x-8$
27. $2(3x+8) -6 =4x+20$
28. $6-x=10+x$
29. $5(x+4)=4(x+2)$
30. $6(x-3)=9(x+12)$

15. FORMULAS

"The difference between the poet and the mathematician is that the poet tries to get his head into the heavens while the mathematician tries to get the heavens into his head." — G.K. Chesterton

There are some times when you have a formula, but it's expressed in terms of the wrong variable. For example, you may find a formula for converting Celcius to Farenheit, when what you want to do is the reverse. At those times, you'll need to solve an equation for one of the variables within it.

The process is actually quite simple: just act as though all the other variables are numbers. Isolate the one you want—consider circling it throughout the problem so that you keep your eye on the right variable.

BE CAREFUL!!! When solving for one variable in terms of others, remember: you can ONLY combine like terms.

Here's the example I mentioned above:

EXAMPLE : F = 9/5 C +32. Solve for C.

We want to isolate the C. The first term to move is the constant: subtract 32 from both sides, giving us F-32 = 9/5C. Next, we want to eliminate that coefficient. We could divide both sides by 9/5, but dividing by a fraction can be messy. It will be easier to just multiply both sides by the reciprocal, 5/9. That gives 5(F -32)/9 = C.

PRACTICE PROBLEMS: Solve each of the following examples for the given variable.

1. C= d for d
2. A=lw for w
3. P = 2l+2w for l
4. A = ½ bh for b
5. V = lwh for w
6. A = 360/n for n
7. S= 180(n-2) for n
8. X=ay+b for y
9. X=-b/2a for a
10. E = 180(n-2)/n for n
11. 2x+y=6x-4y for x
12. 3x+y= 5x-3y for y
13. Feeling brave??? $A = \frac{s^2}{4}\sqrt{3}$ for x

16. INEQUALITIES

Q: Why didn't the quarter roll down the hill with the nickel?

A: Because it had more cents.

As you remember from elementary school, there are times when we need to deal with an inequality instead of an equation. Inequalities deal with one quantity that is larger or smaller than the other, as opposed to an equation where both quantities are equal.

First, let me refresh your memory on the symbols:

SYMBOL	MEANING	EXAMPLE
<	Less than	3 < 6
	Less than or equal to	9 10, 10 10
>	Greater than	8 > 2
	Greater than or equal to	100 90, 100 100
	Does not equal	4 3

To be honest, the vast majority of time when we use an inequality, it will be one of the first 4 of the symbols in that chart.

Now, while you may have learned all sorts of wonderful ways to remember all this (which may or may not have involved an alligator eating your numbers), here's the reality: THE SMALLER OF TWO NUMBERS IS ALWAYS THE ONE ON THE LEFT. That's all there is to it.

PRACTICE PROBLEMS: PLACE EACH GROUP OF NUMBERS IN ORDER FROM SMALLEST TO LARGEST, USING THE FORM

"A < B<C."

1. 4, 46, 20
2. -2, 2, 0
3. -6, -8, -10
4. -3, -3.5, -2.5
5. 0.1, -0.1, -0.001
6. -.4, -.444, -.44
7. -59, -49, -69
8. – ½, -¾, -¼
9. -10, -100, 100
10. 0.1, 0, -0.05

17. THE NUMBER LINE

“Since the mathematicians have invaded the theory of relativity I do not understand it myself anymore.”- Albert Einstein

There are times when we want to see a visual representation of the numbers we’re using in a problem. At those times, it’s helpful to draw a NUMBER LINE. A typical number line includes equal amounts of positive and negative numbers—though that’s not always practical. (For example, if you’re representing the years that your family members were born, I’m pretty sure that the first 1900 or so positive numbers would probably be a waste of space.)

When we plot numbers on a number line, we darken the part of the number line that contains the numbers we want. If it’s a group of isolated numbers—say, the integers—we can simply put a dot on those spots. But if we want to represent an inequality, we darken a whole section of the number line.

If our inequality is either or , we use a dot at the beginning and ending points of the solution set. But if our inequality is either < or >, we use an open dot instead. That open circle represents the idea that we can’t actually name the first (or last) element of the set. (Think about it: what’s the first number greater than 5?? 6?? 5.1?? 5.01?? 5.000001?? You get the idea. That open circle indicates that we can’t name the actual starting place for the set.)

There are some times when it’s simply not practical to show a number line that has zero in the middle of the numbers you’re showing—the numbers are simply too high or too low to make that realistic. In those cases, try to include a few numbers on either side of your solution set.

18. SET NOTATION AND INEQUALITIES

"Women have a passion for mathematics. They divide their age in half, double the price of their clothes, and always add at least five years to the age of their best friend." – Marcel Achard

When you find the solution to an inequality, you need to graph it on a number line. But, after that, you need to express the answer in Set Notation. Set notation begins and ends with parentheses, like these: { }. They're followed with "x:" which translates to "the set of all x's such that…"

Let's say your answer was all x's to the right of 5. In Set Notation, that would be expressed as {x: x>5}

Sometimes your inequality will be the region between two endpoints. In a case like that, your set notation looks like this: {x: left boundary < x < right boundary} For example, if your answer was a pair of solid dots at -2 and at 7, with the region between them shaded, your set notation would be: {x: -2 x 7}

PRACTICE PROBLEMS: GRAPH ON A NUMBER LINE AND EXPRESS IN SET NOTATION… THE SET OF ALL NUMBERS:

1. Less than 4
2. Greater than 28
3. Less than or equal to -5
4. Greater than or equal to -11
5. At least 33
6. At most 56
7. More than 2
8. Smaller than 899
9. Greater than 2 but less than 5
10. Greater than -23 but less than -6
11. Greater than or equal to -3 but less than 12
12. Greater than or equal to 6 but less than 9
13. Greater than 16 but less than or equal to 20
14. Greater than 3 but less than or equal to 5
15. Greater than -5 but less than -4
16. Greater than 0 but less than 1
17. Greater than 3 but less than 6
18. Greater than 12 but less than 23
19. Greater than 100 but less than 105
20. Greater than 205 but less than 215

19. SOLVING INEQUALITIES

"Some people believe in imaginary friends. I believe in imaginary numbers." — R.M. ArceJaeger

Solving inequalities is very much like solving equations, with a few MAJOR differences:

- When you multiply or divide both sides of an inequality by a negative number, you MUST flip the inequality. That means that < becomes > and > becomes <.

- When you have a variable on the right side of an inequality, you'll want to reverse it so the variable is on the left. (So, for example, your answer would be "x < 3" instead of "3 >x") But reversing an inequality is another time when you MUST flip the inequality.

- When you've finished your problem, you put your answer into set notation. Let's say, for example, that your answer to the inequality was " x > 6." Here's how you would write it:

- {x: x> 6}. The parentheses mean that it's a set of numbers. That answer is read as "The set of all x's such that x is greater than 6."

BE CAREFUL!!!! Those rules about flipping the inequality tend to sneak up on you. When dealing with an inequality, you need to always be on the lookout for them. Remember, you flip an inequality when you multiply or divide by a negative number, or when you reverse an inequality.

PRACTICE PROBLEMS: SOLVE, GRAPH ON A NUMBER LINE, AND EXPRESS IN SET NOTATION:

1. X+3>20
2. 4-Y>14
3. 2X 18
4. 5+X 9
5. 2X+3 < 11
6. 2(3X+6) 36
7. 5(2X+4) 80
8. 8X +4 6X-6
9. 7X+15 5X-20
10. 12(X+2) -6 > 6X
11. 2(X+3) > 4X-2
12. 3X+2(X+3) < 7X+10
13. 2(4X-8) +3X > 12X+22
14. 10X – (2X+4) 12X+16
15. 9X-2(3X+12) < 7X+24
16. 15– 3(X+4) 6X -9
17. – (6X-6) < 6X
18. 8X – (4X+24) -36 < 6X+10
19. 12X – (9X+18) > 2X+20
20. 3(X+7) -2(X+4) > 19
21. 5x - 15 >75
22. 42 > 6x+18
23. 15x - 5(2x-7) < 105
24. 3(2x+4) -2(2x+1) > x-14
25. 6(4x+3) > 2(2x+10)

20. ABSOLUTE VALUE EQUATIONS

"Nothing takes place in the world whose meaning is not that of some maximum or minimum." — Leonhard Euler

Absolute value is a mathematical operation. The definition of Absolute Value is the distance from that number to 0 on a number line. Since it's a measure of distance**, Absolute Value can never be negative!!!** It's represented with a pair of vertical lines, like this: -3 | = 3.

When solving an equation involving Absolute Value, here's the process:

- Isolate the part of the equation with the absolute value symbol. Use inverse operations to get that part of the equation on one side of the equal sign by itself.

- Set up two columns. The first is the original problem without any absolute value symbols. The second has the same quanity that was inside the absolute value—don't change anything here—but it's equal to the OPPOSITE of what was on the other side.

- Solve both equations.

- Check for Extraneous Roots. This is kind of strange: sometimes, with particular kinds of equations, you can get do everything right, and get an answer that doesn't check. These are known as Extraneous Roots, and you eliminate them as answers.

- Express any answers that have checked in a solution set.

Example: | x+ 4 | - 5 =7.

x+4 = 12

x+4 = 12   x + 4 = -12

x = 8   x = -16

Check: | 8+4 | -5 = 7 | -16 + 4|- 5 = 7

|12 | -5 = 7 |-12| -5 = 7

12 – 5 = 7   12 – 5 = 7

And, because I KNOW you're wondering, there ARE some practical applications for absolute value: http://mathforum.org/library/drmath/view/57177.html

PRACTICE PROBLEMS: Solve and check each of the following equations:

1. |x+2 |=5
2. |3x|=12
3. |x-4|=8
4. |2x+1|=9
5. |3x-2|=7
6. |x-4|+2 =10
7. |x-6|+5=2
8. |2x+5|-10 =25
9. |x|-4 = 6.5
10. |4x+6|=26
11. |x-3|=2x
12. |x+10|=3x+1
13. |4x-2|+3 =29
14. 2|x-1|= x+4
15. |5x|=x
16. |x+6|-4= 2x
17. |x-8|+2= ½ x
18. |3-x|=x+1
19. 5-|x|=3
20. 10 - |x+1|= 2x
21. |x+16|-8= 2x
22. |x-8|+20= 3 x
23. |10-x|=2x+4
24. 5+|x|=7

21. ABSOLUTE VALUE INEQUALITIES

"Although personally, I think cyberspace means the end of our species." – Michael Crichton

Absolute Value Inequalites combine the ideas from Absolute Value Equations and from regular inequalities.
Here's the process:

- Isolate the Absolute Value, using inverse operations.

- Set up two columns. In the first, rewrite the problem with no absolute value; don't change anything else. In the other column, negate everything after the absolute value, INCLUDING THE INEQUALITY.

- Solve both inequalities.

- Plot them on a number line

- Express the answer in set notation.

Example: $+4 < 13$

$|2x + 3| < 13$

$2x + 3 < 13$	$2x + 3 > -13$
$2x < 10$	$2x > -16$
$x < 5$	$x > -8$

On a number line, both the -8 and the 5 would have open circles, and the region between them would be shaded. The solution set is $\{x: -8 < x < 5\}$

BE CAREFUL!!! When an absolute value inequality is "less than", you'll end up shading part of the number line between two endpoints. When it's "greater than", you'll end up shading both endpoints and the region away from them, so it will be both arrows.

BE CAREFUL!!! When an absolute value inequality is "less than", you'll end up shading part of the number line between two endpoints. When it's "greater than", you'll end up shading both endpoints and the region away from them, so it will be both arrows.

PRACTICE PROBLEMS: Solve, graph on the number line, and express in set notation:

1. $|x|<4$
2. $|x| <6$
3. $|x|> 9$
4. $|x| <3$
5. $|x+2|\ 6$
6. $|2x|>4$
7. $|x-3|<4$
8. $|x+2|+ 3 > 8$
9. $|x-8|+4 > 6$
10. $|2x+1|> 9$
11. $|3-x|<5$
12. $|-x| >4$
13. $2|x|<12$
14. $|x-2|<x$
15. $|x+1| >-x$
16. $|x+10|>2x-4$
17. $|3x|<x+8$
18. $|5x| > x+16$
19. $|6x|< x+24$
20. $|2x+3| >4x-1$
21. $|5x| > x+16$

19. $|4x|< x+16$
20. $|6x+6| >4x-2$

22. VERBAL PROBLEMS: NUMBER PROBLEMS

"Its a mathematical fact that two negatives make a positive. So even under adverse circumstances think positively." — Amit Abraham,

This type of problem asks you to find either a number or a set of numbers which meet particular requirements. Like all other verbal problems, these begin with a LET statement and end with a sentence. They involve translating words into an algebraic equation, then solving that equation.

BE CAREFUL!!! Expressions such as "twice the sum of… " and "half the difference of…" require parentheses.

EXAMPLE: The larger of two numbers is one more than twice the other. Their sum is 37. Find the numbers.

Let x = smaller number

Let 2x+1 = the larger number

$x + 2x + 1 = 37$

$3x + 1 = 37$

$3x = 36$

$x = 12$

Now go back to your LET statements and fill in your answer: The numbers are 12 and 25.

Number problems

1. Six more than twice a number is 38. Find the number.
2. Ten more than half a number is 27. Find the number
3. Five less than one third of a number is 12. Find the number.
4. Twice a number, decreased by 3, is 35. Find the number.
5. Six times a number, increased by 7, is 43. Find the number.
6. Ten less than 4 times a number is 26. Find the number.
7. Fifteen more than 4 times a number is 55. Find the number.
8. Six times a number, decreased by 8, is 46. Find the number.
9. Three times the sum of a number and 6 is 129. Find the number.
10. Five less than 4 times the sum of a number and 8 is 43. Find the number.
11. Twelve more than 3 times the sum of a number and 20 is 216. Find the number.
12. Five less than 3 times the sum of a number and 6 is 184. Find the number.
13. Eight less than three times a number, increased by 4, is 22. Find the number.
14. 100 more than six time the sum of a number and 30 is 340. Find the number.
15. Sixteen less than half the sum of a number and 38 is 52. Find the number.

23. CONSECUTIVE INTEGER PROBLEMS

"I guess I think of lotteries as a tax on the mathematically challenged." – Roger Jones

Some number problems deal with particular numbers: Consecutive Integers, Consecutive Even Integers, or Consecutive Odd Integers. These tend to be easy problems, since the LET statements are determined by which category of problems you're dealing with. Here are the basics:

Type of numbers	Let statements
Consecutive Integers	Let x = 1^{st} X+1 = 2^{nd} X+2= 3^{rd} X+3 = 4^{th}
Consecutive Even Integers	Let x= 1^{st} X+2 =2^{nd} X+4= 3^{rd} X+6 = 4^{th}
Consecutive Odd Integers	Let x = 1^{st} X+2 = 2^{nd} X+4 = 3^{rd} X+6 = 4^{th}

Nope, not a mistake. The LET statements for Consecutive Even and Consecutive Odd integers are exactly the same. It's up to the person making up the problems to ensure that the original x value matches the criteria set up in the problem.

Think about it for a second: if the first even integer is x, why must the next even be x+2??

And if the first odd integer is x, why must the next odd be x+2??

BE CAREFUL!!!! The LET statements for odd integers tend to throw people off, since you're adding an even amount to the last integer to get to the next. You need to pay *attention to these problems!*

PRACTICE PROBLEMS:

1. The sum of three consecutive integers is 33. Find them.
2. The sum of five consecutive odd integers is 27. Find them.
3. The sum of four consecutive even integers is 76. Find them.
4. The sum of 3 consecutive integers is 71. Find them.
5. The sum of three consecutive odd integers is -45. Find them.
6. The sum of 4 consecutive even integers is 0. Find them.
7. The sum of 3 consecutive even integers is -36. Find them.
8. The sum of 3 consecutive integers is 3. Find them.
9. The largest pf 4 consecutive integers is 4 times the smallest. Find all 4 integers.
10. If the largest of 3 consecutive odd integers is increased by twice the smallest, the result is 37. Find all three.
11. Find two consecutive even integers such that twice the smaller diminished by twenty is equal to the larger.
12. Find 3 consecutive odd integers whose sum is 171.
13. Find 3 consecutive integers such that the larger is twice the smaller.
14. Find two consecutive even integers whose sum is 70
15. Find 3 consecutive odd integers whose sum is 3.
16. Find 5 consecutive even integers whose sum is 0.

17. If the largest of 4 consecutive even integers is increased by the smallest, the result is 34. Find all four.

18. The sum of 5 consecutive integers is 0. Find them.

9. Find 3 consecutive odd integers such that twice the sum of the largest and smallest is 70.

20. Find 3 consecutive even integers such that half the sum of the largest and smallest is 47.

21. Find 3 consecutive integers such that twice their sum, increased by 8, is 74.

22. What two consecutive odd integers have a sum of 76?

23. Find three consecutive even integers such that the sum of the second and the third is equal to the first

24. Find three consecutive odd integers such that the sum of the first and third is equal to the sum of the second and 7.

25. Find three consecutive odd integers such that eight more than the sum of the first two is equal to eleven less than three times the third.

26. Find two consecutive even integers such that twice the smaller diminished by twenty is equal to the larger.1. If the largest of 4 consecutive even integers is increased by the smallest, the result is 34. Find all four.

27. The sum of 5 consecutive integers is 0. Find them.

28. Find 3 consecutive odd integers such that twice the sum of the largest and smallest is 70.

29. Find 3 consecutive even integers such that half the sum of the largest and smallest is 47.

30. Find 3 consecutive integers such that twice their sum, increased by 8, is 74.

31 What two consecutive odd integers have a sum of 76?

32. Find three consecutive even integers such that the sum of the second and the third is equal to the first

33 Find three consecutive odd integers such that the sum of the first and third is equal to the sum of the second and 7.

34 Find three consecutive odd integers such that eight more than the sum of the first two is equal to eleven less than three times the third.

35. Find three consecutive odd integers such that the sum of the first and third is equal to the sum of the second and 7.

36 Find three consecutive odd integers such that eight more than the sum of the first two is equal to eleven less than three times the third.

24. VERBAL PROBLEMS: MONEY PROBLEMS

"99 percent of all statistics only tell 49 percent of the story."
— Ron DeLegge II

These problems deal with something you're already good at: the value of money. Most involve the idea of taking a handful of change, counting the number of each type of coin, and determining the total value. (Think about it: if you had 12 dimes, you wouldn't count 10, 20, 30… right?? You would say 12 (.10) = 1.20 .That's the idea we'll use in Money problems. Sometimes we use stamps instead of money, since the problem can be set up to give the stamps any value the writer wishes.)

Money problems use a chart to organize information. The headings on the chart are:

Type of coin	# of coins	value	total
quarter	10	25	250

The last column is the product of the 2 that precede it.

In the example above, I wrote the value of a quarter as 25 cents, instead of 0.25. You may find that it's easier this way, since you don't need to use decimals. But be careful!!! Make sure the total amount of money is also expressed in cents; you can't mix cents and dollars in the same problem!!!

BE CAREFUL!!!! As with other problems, your answers should make sense!! If the problem asks how many quarters Tim has, you can't get an answer like -4 or 6.5. Your answer must be a positive whole number. If your answers don't make sense, you should realize there's an error somewhere.

PRACTICE PROBLEMS

1. Michael has some coins in his pocket consisting of dimes, nickels and pennies. He has 2 more nickels than dimes, and 3 times as many pennies as nickels. How many of each coin does he have if the total value is 52 cents?

2. A coin collector has a collection of silver coins worth $205. There are 5 times as many quarters as half-dollars and 200 fewer dimes than quarters. How many of each coin did the collector have?

3. A collection of coins has a value of 64 cents. There are 2 more nickels than dimes and 3 times as many pennies as dimes. How many of each coin are in the collection?

4. Tanya has ten bills in her wallet. She has a total of $40. IF she has one more $5 than $10 bills, and 2 more $1 than $5 bills, how many of each does she have?

5. Mario bought $21.44 worth of stands at the post office. He bought 10 more 4-cent stamps than 19-cent stamps. The number of 32-cent stamps was three times the number of 19-cent stamps. He also bought 2 $1 stamps. How many of each kind of stamp did he buy?

6. A clerk at the Kellenberg Kafe receives $15 in change for her drawer at the start of each day. She receives twice as maney dimes as 50-cent pieces, and the same number of quarters as dimes. She has twice as many nickels as dimes and a dollar's worth of pennies. How many of each coin does she receive?

7. A collection of 36 coins consists of nickels, dimes, and quarters. There are 3 fewer quarters than nickels and 6 more dimes than quarters. How many of each type are in the collection?

8. Brian had 5 times as many quarters as dimes. If the total value of her coins was $16.20, how many of each coin did he have?

9. Jenny received $6.10 in tips. All of her tips were in quarters, dimes and nickels. There were 5 fewer dimes than quarters and 7 fewer nickels than dimes. How many of each type of coin were there?

10. Grant's change jar contained $8.80 in quarters, dimes and nickels. There were two more than 5 times as many nickels as quarters and four fewer than twice as many dimes as quarters. How many of each type of coin were there in the change jar?

11. A bank teller had 125 bills in $10 and $20 to start the day. If the total in her register was $16.50, how many of each bill did the teller start with?

12. Gary has a coin jar. The jar currently contains 53 coins consisting of nickels, dimes and quarters. The number of quarters is equal to the number of nickels. The total value of the coins is $6.80. How many of each type of coin does Grant have?

13. In the 19th Century, the U.S. minted two-cent coins and three-cent coins. Charlie has three times as many three-cent as two-cent coins. The face value of the coins is $1.10. How many of each type of coin does Charlie have?

14. A child's bank has a collection of nickels, dimes and quarters with a total value of $4.15. There are 4 times as many dimes as nickels, and 39 coins in all. How many of each type of coin are in the bank?

25. VERBAL PROBLEMS: MOTION PROBLEMS

"Every formula which expresses a law of nature is a hymn of praise to God." — Maria Mitchell

Every comedian who ever made fun of high school math begins his spiel the same way: "Two trains leave a station…" http://tvtropes.org/pmwiki/pmwiki.php/Main/TrainProblem

Well, here's where they got their material. We're about to learn how to solve problems that involve travel.

Like Money problems, these also involve a chart. Think about it for a minute: if you drive 3 hours, averaging 50 miles per hour, then how far will you drive? 150 miles, right?? That's because of the formula (RATE)(TIME)= DISTANCE. That formula will form our chart:

Person	rate	time	distance

As was the case with coin problems, the last column is the product of the 2 columns that precede it.

BE CAREFUL!!! Since the rate will usually be in "mph"—miles per hour--, your time needs to also be in terms of hours. So, for example, if the time spent driving is 1 hour and 15 minutes, you'll need to covert that to 1.25 hours. (Stop. Think for a second. Why is it 1.25 hours and not 1.15 hours???)

Also, realize that these problems sometimes use the Metric System—that does NOT make things any more difficult. A Kilometer (km) is almost 2/3 of a mile. So a car that is travelling 60 mph is going approximately 96.5 km/hour.

BE CAREFUL!!!! As with other problems, your answers should make sense!! A plane that is going 30 mph will fall out of the sky. A person who is running 50mph will collapse of a heart attack. If your answers don't make sense, you should realize there's an error somewhere.

Practice Problems

1. A freight train starts from LA and heads for Chicago at 40 mph. Two hours later, a passenger train leaves the same station for Chicago traveling at 60mph. how long will it be before the passenger train overtakes the freight train?

2. A car leaves San Francisco for LA traveling an average of 70 mph. At the same time, another car leaves LA for San Francisco traveling at 60mph on the same road. If it is 520 miles between San Francisco and LA, how long before the two cars meet?

3. Two planes leave JFK at 10 AM, one heading for Europe at 600 mph and one heading in the opposite direction at 150 mph… obviously NOT a jet! At what time will they be 900 miles apart? How far has each traveled?

4. Mr. Smith makes a business trip from his house to Montauk in 2 hours. One hour later, he returns home in traffic at a rate 20 mph slower than his rate going. If Mr. Smith is gone a total of 6 hours, how fast did he travel on each leg of the trip?
5. Sarah leaves Seattle for NY in her car, averaging 80mph across open country. One hour later, a plane leaves Seattle for NY following the same route and flying 400 mph. How long will it be before the plane overtakes Sarah's car?

6. Two planes leave Kansas City at 1 PM. Plane A travels east at 450 mph and plane B heads west at 600 mph. At what time will the two planes be 2100 miles apart?

7. Tim leaves home for Skedunk, 400 miles away. After 2 hours he has to reduce his speed by 20mph due to rain. If he takes 1 hour for lunch and gas, and reaches Skedunk 9 hours after leaving the house, what was his initial speed?

8. A destroyer traveling at 40 knots and a battleship traveling at 30 knots left at the same naval base at the same time and sailed in opposite directions. In how many hours were the ships

9. At 8am, 2 cars started from the same place, one traveling north at 35 mph and the other traveling south at 40 mph. At what time were they 300 miles apart?

10. A destroyer traveling at 40 knots and a battleship traveling at 30 knots left at the same naval base at the same time and sailed in opposite directions. In how many hours were the ships 350 miles apart?

11 2 planes left an airport at the same time, one flying east at 180 mph and the other flying west at 320 mph. For how many hours of traveling time were they no more than 1,000 miles apart?

12. At 8am, 2 cars started from the same place, one traveling north at 35 mph and the other traveling south at 40 mph. At what time were they 300 miles apart?

13. A northbound & a southbound train left Tootleville station at the same time. The southbound train was traveling 20 mph faster than the northbound train. After an hour, they were not more than 100 miles apart. What is the maximum possible speed for the northbound train?

14. A salesman made a trip of 375 miles by bus and train. He traveled 3 hours by bus and 4 hours by train. If the train averaged 15 mph more than the bus, find the rate of each.

15 A motorist made a trip of 275 miles in 8 hours. Before noon she averaged 40 mph and after noon she averaged 25 mph. At what time did she begin her trip and when did she end it?

16. A freight train left a station and traveled at 30 mph. 2 hours later, an express train left the same station and traveled in the same direction at 50 mph. In how many hours did the express train overtake the freight train?

17. Two cyclists start at the same corner and ride in opposite directions. One cyclist rides twice as fast as the other. In 3 hours, they are 81 miles apart. Find the rate of speed of each cyclist.

18. A jogger started running at an average speed of 6 mph. Half an hour later, another runner started running after him from the same place at an average speed of 7 mph. How long will it take for the runner to catch up with the jogger?

19. A 555-mile, 5 hour trip on the Autobahn was drive at two speeds. The speed of the car was 105 mph for the first part of the trip, and the average speed was 115 mph for the second part. How long did the car drive at each speed?

20. Andy and Beth are at opposite ends of an 18-mile country toad with plans to leave at the same time running toward each other to meet. Andy runs 7mph while Beth runs 5 mph. How long after they begin will they meet?

21. A car and a bus set out at 2 pm from the same spot, headed in the same direction. The average speed of the car is double the average speed of the bus. After 2 hours, the car is 68 miles ahead of the bus. Find the rate of each.

22. A pilot flew from one city to another averaging 150 mph. Later, it flew back to the first city averaging 100 mph. The total flying time was 5 hours. How far apart are the cities

26. VERBAL PROBLEMS: LEVER PROBLEMS

Mathematics is the art of explanation." – Paul Lockhart,

Lever problems involve a tool that you're very familiar with, since a See Saw is a type of lever. The basic idea is that 2 weights are on opposite sides of a balancing point called the Fulcrum. Think about the last time you were on a see-saw: where did the heavier person sit? If you wanted to balance that see-saw, the heavier person needed to sit closer to the fulcrum, while the lighter person sat further away.

The formula for levers is:

(1st weight)(1st distance) =(2nd weight)(2nd distance.)

27. VERBAL PROBLEMS: PULLEY PROBLEMS

Dear Math, please grow up and solve your own problems, I'm tired of solving them for you.

Like Lever problems, Pulleys also involve a tool that you're already familiar with. In this case, it's the chain on your bicycle. Pulleys involve two circles, held together by a rope or chain. For each complete turn made by the larger pulley, the smaller pulley needs to make more than one turn. The formula for pulleys is: (1^{st} diameter)(1^{st} speed) = (2^{nd} diameter)(2^{nd} speed.)

The speed at which pulleys turn is measured in Rotations per Minute: rpm's.

PRACTICE PROBLEMS:

1. Mary weighed 120 pounds and sat on one end of a seesaw 8 feet from the center. Jim weighed 160 pounds (what were those 2 doing on a seesaw??) How far from the fulcrum must Jim sit in order to balance Mary?

2. A pulley with a radius of 6 inches rotates at 120rpm. It is attached to a pulley with a radius of 8 inches. How fast does the larger pulley rotate?

3. A weight of 60 pounds rests on the end of an 8 foot lever and is 3 feet from the fulcrum. What weight must be placed on the other end of the lever to balance the 60-lb weight?
4. A pulley with a diameter of 12 cm rotates at 640rpm. It is attached to a pulley with a diameter of 6cm. How fast does the smaller pulley rotate?

5. Kerri weighs 84 pounds and sits 3 feet from the fulcrum of a seesaw. Her youngest sister, Katie, weighs 42 pounds. How far from the fulcrum should Katie sit?

6. A pulley with a diameter of 24cm rotates at a speed of 400rpm. It is attached to a pulley rotating as a speed of 600rpm. What is the radius of the second pulley?

7. Frank weighs 96 pounds and sits 3 feet from the fulcrum of a seesaw. His brother weighs 144 lbs; how far from the fulcrum should the brother sit to balance the seesaw?

28. VERBAL PROBLEMS: ANGLE PAIRS *

Math is fun, it teaches you life and death information, like when you're cold, you should go to a corner since it's 90 degrees there. Anonymous

Next year, in Geometry, you'll spend a lot of time working with angles. As you may recall from elementary school, an angle is the union of two rays sharing a common endpoint. This year, we're going to work with three particular types of pairs of angles.

Supplementary Angles are two angles, the sum of whose measures is 180 degrees. The angles can be next to each other, like this:

Or they can be just any two angles whose sum is 180 degrees, like a 40 degree angle and a 140 degree angle.

Complementary Angles are two angles, the sum of whose measures is 90 degrees. Like supplements, you can either be given a picture or information about the measures of the angles. If it's in pictoral form, there will be a small box where the right angle is:

Vertical Angles are angles which share a common endpoint and whose sides are opposite rays. They're a lot easier to spot than they are to define… they're angles "across the X" from each other. VERTICAL ANGLES ARE EQUAL

X

In the picture above, angles 1 and 2 are vertical angles, which means they have the same measure. Algebraically, that means you would set the measure of <1 equal to the measure of <2, and then solve the resulting equation.

Practice Problems

1. Two angles are complements. One is 20 degrees more than the other. Find both angles.

2. Two angles are supplements. The larger is 10 more than 4 times the smaller. Find both angles.

3. Two angles are complements. One is twice the measure of the other. Find both angles.

4. Two angles are supplements. They're equal in measure. Find both angles.

5. The complement of an angle is 10 degrees more than five times the angle. Find both angles.

6. The supplement of an angle is three times the complement of the angle. Find the angle.

7. The complement of an angle is 32 degrees more than the angle. Find both angles.

8. The supplement of an angle is 12 less than twice the angle. Find both angles.

29. VERBAL PROBLEMS: TRIANGLE PROBLEMS

An adult is a person who no longer grows in height, but instead grows in length and width. Anonymous

Triangles are another of the geometric topics you'll see just a little of this year. There are a few rules of triangles that particularly lend themselves to Algebra:

- The sum of the angles of a triangle is always 180 degrees. That one you're probably pretty familiar with.

- The sum of two sides of a triangle is greater than the 3rd side. Think about it for a second. Let's pretend that your older brother has his license, and drove you to school this morning. I don't' care how fast he drove (well, as another driver on the roads this morning, actually I DO care, but that's not the point here.) If he detoured to 7-11 or Dunkin Donuts or Starbucks on the way to school, he drove more miles than if he had come straight here. The shortest distance between two points (here the points are home and school) is a straight line; the sum of the other 2 sides of a triangle will always be greater than the 3rd, direct, side

- A triangle always has the same number of congruent (that means equal in measure) sides as it has congruent angles.

Let's get back to that first rule. As you probably learned in elementary school, there are a number of different ways to classify triangles. We can do it by their sides:

Number of congruent sides	Name
0	Scalene
2	Isosceles
3	Equilateral

Or by their angles:

Types of angles	Name
All less than 90 degrees	Acute
One equal to 90 degrees	Right
One greater than 90 degrees	Obtuse
All equal to each other	Equiangular

Once we know the measures of the angles of a triangle, we can classify it both ways. For example a triangle with angles measuring 30, 60 and 90 degrees respectively is a Scalene Right triangle.

PRACTICE PROBLEMS:

1. In triangle ABC, <B is 70 degrees more than <A. <C is the sum of the other two angles. Find all 3 angles and classify the triangle.

2. In Triangle ABC, <B is 20 less than <A.   <C is 40 more than the sum of the other two angles. Find all 3 angles and classify the triangle.

3. In Triangle ABC, <B is 35 degrees less than <C. <A is 9 less than 5 times <B. Find all 3 angles and classify the triangle.

4. In triangle ABC, <A = <B. <C is twice as large as <A. Find all three angles and classify the triangle.

5. In Triangle ABC, <B is 20 degrees more than <B. <C is 30 degrees more than 3 times <B. Find all three angles and classify the triangle.

6. In Triangle ABC, <A is three times <C and <B is twice as large as <C. Find all three angles and classify the triangle.

7. Two angles of a triangle are congruent. The third angle is twice either of the smaller angles. Find all three angles and classify the triangle.

8. In Triangle ABC, <B is 30 more than <C. <A is 3 times <C. Find all three angles.

30. VERBAL PROBLEMS: PERIMETER PROBLEMS

Finding a treasure is like working on algebra equations, all you have to do is find the X. -

This is the last of the Geometry topics we'll be covering for a while. As you may remember, the perimeter of a figure is the sum of the measures of its sides. (If your figure happens to be a rectangle, you may recall either the formula P = 2l + 2w, or P = 2(l + w). And if it's a square, you may recall it as P = 4s. But in general, it's probably easiest to remember as the sum of the sides.)

BE CAREFUL!!! In perimeter problems, you should always be able to account for each one of the sides. So, for example, if your figure has 5 sides, there should be 5 algebraic expressions added together. Sure, some of them can be equal. But always check to ensure that you've included all the necessary sides.

Sometimes, instead of giving you the number of sides contained in the polygon, you'll be given its name. Here are some you'll want to know:

Number of sides	Name
3	triangle
4	quadrilateral
5	pentagon
6	hexagon
8	octagon
9	nonagon
10	decagon

PRACTICE PROBLEMS:

1. The length of a rectangle is twice its width. The perimeter is 138 feet. Find its dimensions.
2. The length of a rectangle is 4 less than 3 times its width. The perimeter is 224 feet. Find its dimensions.

3. The 1st side of a triangle is 2 inches less than twice the 2nd side. The 3rd side is 10 inches longer than the 2nd side. If the perimeter si 12 feet, find the length of each side.

4. The length of a rectangular soccer field is 10 feet more than twice the width. The perimeter is 320 feet. Find the dimensions.

5. A farmer wishes to fence a rectangular area behind his barn. The barn forms one end of the rectangle, and the length of that side is 3 times the adjacent side. If the farmer has 140 feet of fencing, what are the dimensions of his rectangle?

6. The length of a room is 8 feet more than twice the width. If it takes 124 feet of molding to go around the perimeter of the room, what are the room's dimensions?

7. One side of a triangular garden is 2 ft. longer than another side. The 3rd side is 4 feet long. What are the maximum lengths of the other 2 sides if the perimeter of the garden can be no longer than 12 feet?

8. A rectangle has a length which is twice its width. If the length is increased by 4, and the width is decreased by 8, a new rectangle is formed. The new rectangle has a perimeter of 46. Find the original dimensions.

9. The perimeter of a rectangle is 144 feet. The length is 3 times the width. Find the dimensions of the rectangle.

10. The length of a rectangle is 5 feet more than twice the width. Its perimeter is 70 feet. Find the dimensions.

11. In a rectangle, the length is 2 feet less than twice the width, and the perimeter is 2 feet more than seven times the width. Find the dimensions.

12. In a triangle, one side is 6 feet more than the shortest side. The other side is 5 feet less than twice the shortest side. The perimeter is 33 feet. Find the length of each side.

13. In an isosceles triangle, two of the sides have the same length. The other side is 20 cm less than twice the length of one of the equal sides. The perimeter is 46 cm. Find the measure of each side.

14. In a triangle, the lengths of the sides are consecutive integers. The perimeter is 11 more than twice the shortest side. What is the perimeter?

31. POLYNOMIALS AND DEGREE

In Math class we learned more about algebra today, such as X+10=Y should I care?

A Polynomial is an algebraic expression composed of one or more terms. Some particular polynomials have names, as shown below:

Number of terms	Name	How to remember	Example
1	Monomial	"mono" means "one"	$6xy^2$
2	Binomial	"bi" means 2—think "bicycle"	$x^2 - 3y^3$
3	Trinomial	"tri" means 3—think "tricycle"	$x^2 - 2x + 10$

If a polynomial has more than 3 terms, we just call it a polynomial.

The Degree of a monomial is the sum of the exponents of its variables. For example, the degree of $4x^3$ is 3, because its only exponent is 3. But the degree of $6xy^4$ is 5, because the x is understood to be to the first power, the y is to the 4^{th}, and 1+4=5.

To find the degree of a polynomial, compare the degrees of each of its terms and choose the highest.

Very often, we'll want our polynomials in Standard Form. That means we'll want to arrange its terms so that the highest degree comes first, and the degrees of the remaining terms count down, or descend.

Finally, the Leading Coefficient is the coefficient of the degree term.

PRACTICE PROBLEMS**:** Classify each polynomial (monomial, binomial, trinomial or polynomial) find its degree, its leading coefficient and put into standard form:

1. $7x^3 + 4x^2 - 6x^5 + -5x8$
2. $4x+x^2 -9$
3. x
4. $10 -6x^3 -8x$
5. $3x^3y$
6. $4x -5x^2$
7. $-9x^5 +8x -3x^4$
8. $12x^4 -5x^7$
9. $10+x$
10. $3x$

32. ADDITION AND SUBTRACTION OF POLYNOMIALS

Three statisticians go out hunting together. After a while they spot a solitary rabbit. The first statistician takes aim and overshoots. The second aims and undershoots. The third shouts out "We got him!"

Addition of polynomials is something you've actually already done. It's just a matter of combining like terms.

EXAMPLE: Add $(4x^3 - 6x^2 + x - 3) + (2x^3 - 5x^2 + 6x + 11)$

ANSWER: Just combine the like terms… remember, that means you combine the coefficients, but don't change any exponents. You sum should be $6x^3 - 11x^2 + 7x + 8$.

To subtract polynomials, remember the rule about a negative sign outside parentheses: insert a 1 between the negative sign and the parentheses, then distribute the -1 and combine like terms.

EXAMPLE: Subtract $(4x^2 - 5x + 2) - (2x^2 - 3x + 7)$

ANWER: When we distribute the -1, we're left with $4x^2 - 5x + 2 - 2x^2 + 3x - 7$

Combining like terms gives is $2x^2 - 2x - 5$

PRACTICE PROBLEMS: SIMPLIFY :

1. $(A+2B+C) + (4A-4B+C)$
2. $(2x+8y-z) + (2x+4y-z)$
3. $(5a-b) + (6a+2b) + (a+b)$
4. $(3x-6y) + (8y-2x) + (9x-8y)$
5. $(x^2+3x-7) + (5x^2-7x+9)$
6. $(3x^2-4x+1) + (5x^2-2x+3)$
7. $(2x^2+5x+1) + (4x^2-3x+1)$
8. $(5x+7x^2-4) + (10+3x-x^2)$
9. $(6x-y) - (4x+2y)$
10. $(3x+4y) - (12y-3x)$
11. $(2x^2+4x+12) - (x^2+6x-1)$
12. $(5c^2+3c-3) - (2c^2+3c-7)$
13. $(12x^2-5x-3) - (3-5x-12x^2)$
14. $(4x^2+8x+4) - (3x^2+8x+10)$
15. $(16x^2-10x+4) - (2x^2+7x+1)$
16. $(33x^2-7x) - (15x^2-4x-2)$
17. $(2x^2+5x+4) - (4+5x+2x^2)$

33. PROPERTIES OF EXPONENTS

There was a statistician that drowned crossing a river… It was 3 feet deep on average.

As you should remember, an exponent tells you how many times the base was multiplied by itself. As a result, there are some basic rules of exponents:

- To multiply powers of the same base, ADD THE EXPONENTS BUT LEAVE THE BASE ALONE. **For example, $(x^2)(x^5) = x^7$** because now 7 x's are multiplying each other.

- To divide powers of the same base, SUBTRACT THE EXPONENTS, BUT LEAVE THE BASE ALONE. **For example, $x^8 \div x^5 = x^3$** because the first 5 x's cancel each other out.

- To raise a power to a power, MULTIPLY THE EXPONENTS BUT LEAVE THE BASE ALONE. **For example, $(x^4)^3 = x^{12}$** because it's $(x^4)(x^4)(x^4)$ and the exponents add up to 12.

- ANYTHING to the zero power is 1. The only way to get an exponent of zero is to divide something by itself—and subtract the exponents and get zero. But anything divided by itself is 1.

- Negative exponents mean you should TAKE THE RECIPROCAL OF THE BASE. The way to get a negative exponent is to have more of the base on the bottom than on the top. So more of those terms end up in the denominator. **For example, $x^2 \div x^7 = x^{-5}$.** When you cancel the 2 x's on the top with 2 of the ones on the bottom, you're left with 5 in the bottom—or $1/x^5$

PRACTICE PROBLEMS: Simplify each problem:

1. $(x^3)(x^8)$
2. $(x^4)(x^3)(x^2)$
3. $(x^{11})(x^{22})$
4. $(x^2)(x^3)(x^4)$
5. $(x^3y^5)(x^4y^7)$
6. $(x^6)\div(x^2)$
7. $(x^{10}) \div(x^5)$
8. $(x^{20}) \div(x^4)$
9. $(x^{12}y^6)\div(x^3y^3)$
10. $(x^{12}y^{15})\div(x^2y^4)$
11. $(x^3)^4$
12. $(x^5)^6$
13. $(x^3y^6)^2$
14. $(x^5y^6z^7)^8$
15. x^{-4}
16. $(x^2y^{-3})^{-2}$
17. $(2x^4y^{-3}z^{-5})^{-3}$
18. $(5x^{-6}y^3z^{-6})^{-2}$
19. $(10x^4y^3z^{-5})^{-2}$
20. $(4x^{-1}y^5z^{-3})^{-2}$
21.

34. SCIENTIFIC NOTATION

Q: Why didn't the quarter roll down the hill with the nickel?

A: Because it had more cents.

Sometimes mathematicians and scientists need to deal with very large or very small numbers. When that happens, it can be particularly annoying—and unnecessary—to actually write out all the digits of those number. This is especially true when those numbers begin or end with a lot of zeros.

As a result, we rely on **Scientific Notation** to make the process easier. Here are the rules:

- Move the decimal point so that the number is now a number between 1 and 9.99.

- Write “x 10” and determine the exponent of the 10. We use the “x” sign to symbolize multiplication (even though we generally don’t do that in algebra) because these problems always involve decimal points, and the alternative is simply too confusing.)

- If you started off with a large number, your exponent will be positive. If you started off with a small number, your new exponent will now be negative.

- Count the number of places you moved your decimal. That’s the new exponent.

Example 1: 920,000,000 = 9.2×10^7

Example 2: 0.000 000 000 12 = 1.2×10^{-10}

To multiply or divide in scientific notation, simply work with the decimal part first, and the exponents next.

Example 3: $(3.1 \times 10^4)(5.0 \times 10^3) = 15.5 \times 10^7$

PRACTICE PROBLEMS: Express in Scientific Notation:

1. 470,000
2. 2,310,000
3. 431,000,000
4. 5,000,000,000
5. 3 million
6. 45,000,000
7. 0.000 000 023
8. 0.000 000 000 01
9. 0.000 000 12
10. 0.000 064
11. 43,000,000
12. 1,234,000,000
13. 140,000,000
14. 0.000 000 009
15. 0.000 000 000 017

EXPRESS IN STANDARD NOTATION:

16. $3.4 \text{ X } 10^4$
17. $7.11 \text{ X } 10^5$
18. $6.8 \text{ X } 10^7$
19. $2.67 \text{ X } 10^5$
20. $9.2 \text{ X } 10^8$
21. $1.4 \text{ X } 10^{-3}$
22. $2.7 \text{ X } 10^{-6}$
23. $6.87 \text{ X } 10^{-4}$
24 $5.43 \text{ X } 10^{-7}$
25. $8.3 \text{ X } 10^{-5}$
26. 5.7×10^{-5}
27. 4.02×10^{-3}
28. 6.004×10^{-2}
29. 5.09×10^{-8}
30. 6.22×10^{-6}

35. MULTIPLYING BY A MONOMIAL

Q: How do you make seven an even number?

A: Take the "s" out!

When a monomial multiplies other terms—either another monomial or a larger polynomial, it relies on the rules for exponents we covered a few pages back. Basically, there are a few rules:

- Multiply the coefficients.
- For each variable, add the exponents.
- If you're multiplying a monomial by a polynomial, remember to distribute the monomial.

Example 1: $(12x^4y^3)(4x^3y^7) = 48x^7y^{10}$

Example 2: $3x^5(6x^3 + 10x^2 - 7x - 1) = 18x^8 + 30x^7 - 21x^6 - 3x^5$

PRACTICE PROBLEMS: FIND EACH PRODUCT:

1. $x(14x^2)$
2. $(4x^2)(3x)$
3. $8x(7x^4)$
4. $(-21x^4)(3x^{10})$
5. $(25x^5y)(4x^6y^3)$
6. $(9x^3)(-9x^5)$
7. $(15x^4)(10x^{12})$
8. $(12x^3)(-2x^2)$
9. $(2x)(3x^4)(5x^4)$
10. $(4x^4)(-7x^2)(3x^5)$
11. $xy(5x^2y^4)$
12. $(2x^3y^5)(-8x^6y^9)$
13. $x(2x-5)$
14. $x(x^2+4x+3)$
15. $2x^2(x^2+4x+5)$
16. $3x^4(x^5-4x^3+2x-7)$
17. $6x^6(4x^6-9x^5+8x^3-x^2+1)$
18. $14xy^2(3x^2-xy+4y^3)$
19. $16xy(4x^2y^5 - 7xy^3+6x^5y)$
20. $17xyz(4x^2y^5z^4 + 2x^6y^7z^2)$
21. $(4xyz^2)(3xy)$

36. MULTIPLYING BINOMIALS

Q: Why should the number 288 never be mentioned?

A: It's two gross.

If you can multiply monomials, then you can multiply any other sort of polynomials. But there are 3 different methods commonly used to multiply binomials; I wanted to briefly explain each of them:

FOIL is an acronym used sometimes to multiply two binomials:

- First: multiply the first term in each set of parentheses
- Outer: multiply the very first term by the very last term
- Inner: multiply the two middle terms
- Last: multiply the last term in each set of parentheses
- Then combine like terms

Example: (x+2)(x+3):

- **F: $x(x) = x^2$**
- **O: $x(3) = 3x$**
- **I: $2(x) = 2x$**
- **L: $2(3) = 6$**

Answer: x^2+5x+6

Another common method is the box method.

- Place the first binomial on the top of a box, with each term in its own column.
- Place the second binomial on the side of the same box, with each term in its own row.
- For each of the interior boxes, multiply the row heading by the column heading.
- Combine like terms

The same Example, done by the box method:

	x	+2
x	x^2	+2x
+3	+3x	+6

Answer: $x^2+ 5x + 6$

But to be honest, the explanation I personally prefer is to simply distribute one term at a time. You already know that anything multiplying a set of parentheses is to distribute to all the interior terms. So simply distribute one term at a time. The beauty of this explanation is that it's not new; you already know how to do it this way.

The same example, one more time:

(x+2)(x+3)

Distribute the x: $x^2 + 3x$

Distribute the +2: $+2x + 6$

Answer: $x^2+ 5x +6$

PRACTICE PROBLEMS: FIND THE PRODUCT, USING WHICHEVER METHOD YOU CHOOSE:

1. (x + 3)(x + 2)
2. (x - 4)(x +6)
3. (x- 3) (x – 4)
4. (x - 10)(x - 2)
5. (x -13)(x -1)
6. (x + 12) (x -5)
7. (x – 4)(x -15)
8. (x-5)(x+8)
9. (xy+2)(xy-7)
10. (xy+9)(xy+9)
11. (x+4)(x+4)
12. (x-6)(x-6)
13. (2x+3)(x -9)
14. (3x+ 4)(2x + 4)
15. (5x+2)(6x -5)
16. (5x+3)(6x-7)
17. $(x^2-4)(x^2+2)$
18. (2x+3)(2x-3)
19. (5x+4)(5x-4)
20. (2x+9)(2x-9)
21. $(x-7)^2$
22. $(2x+5)^2$
23. What pattern do you see in problems # 18-20?
24. How else could problems # 10-12 have been written?

37. MULTIPLYING POLYNOMIALS

Q: Why don't you do arithmetic in the jungle?

A: Because if you add 4+4 you get ate!

Multiplication of polynomials is simply a matter of distributing, one term at a time, from the first parentheses to the second.

BE CAREFUL!!! Before you combine any like terms, the number of terms you should have is the product of the number of terms in the first polynomial times the number of terms in the second. So if you're multiplying 3 terms by 4 terms, you should start off with 12 terms before you combine anything. Get into the habit of checking to ensure that you didn't miss one.

Example: $(x+5)\ (x^2 - 6x + 4)$

ANSWER: Distribute the x: $x^3 -6x^2+4x$

Distribute the +5: $+5x^2 -30x +20$

Combine like terms: $x^3 -1x^2-26x +20$

PRACTICE PROBLEMS: FIND EACH PRODUCT

1. $(x + 2)(x^2 + 3x + 4)$
2. $(x + 4)(x^2 - x + 6)$
3. $(x - 7)(x^2 + 6x - 4)$
4. $(x - 5)(x^2 - 10x - 1)$
5. $(2x + 1)(x^2 - 8x - 3)$
6. $(4x + 2)(x^2 - 12x + 3)$
7. $(5x - 5)(x^2 - x + 6)$
8. $(6x + 2)(x^2 - 7x + 9)$
9. $(3x + 7)(x^2 + 2x + 6)$
10. $(6x - 2)(x^2 + 4x + 5)$
11. $(3x + 2)(2x^2 + 4x - 5)$
12. $(6x - 3)(4x^2 - 10x + 12)$
13. $(7x + 2)(3x^2 - 5x + 9)$
14. $(8x + 7)(4x^2 - 6x + 11)$
15. $(9x + 2)(8x^2 - 7x + 6)$
16. $(x^2 + 4x + 5)(x^2 + 6x + 3)$
17. $(2x^2 + 8x + 9)(x^2 - 4x + 12)$
18. $(x^2+ 5x + 6)^2$
19. $(2x^2+ 7x - 1)^2$
20. $(x^2+ 3xy- y^2)^2$

38. DIVISION OF POLYNOMIALS

Q: Where do math teachers go on vacation?

A: To Times Square.

OK, hang onto your hats… this one can be confusing!!

Division of polynomials is very much like long division of numbers. Before we even begin, let's go over some key words from that process. Let's use the example 10÷ 2 = 5 as our sample:

The Dividend is the number you're starting off with, in this case the 10. The Dividend is the number that goes INTO the division symbol(That's the little "house" you use to divide.)

The Divisor is the number you're dividing into the dividend, in this case the 2. The Divisor goes OUTSIDE the division symbol.

The Quotient is the answer to a division problem, in this case the 5. The Quotient goes on top of the division symbol.

Here's the process:

- Do yourself a favor and make sure both the dividend and divisor are in standard form—that means the exponents count down.

- Place the Dividend into the division symbol. Make sure you leave empty space if there are any missing terms—say, if the problem goes from x^4 to x^2 without any x^3 terms.

- Place the Divisor outside the division symbol.

- Divide the 1st term of the Dividend by the 1st term of the divisor. The quotient goes on top. Unlike arithmetic division, it's not important to line up the quotient over any particular terms in the dividend.

- Anything that goes on top distributes to the divisor. Place the product inside the division symbol, under the dividend.

- Change all the signs of the bottom row.(That's the product you just got by multiplying). Add what's left. Your first terms should cancel.

Repeat the last 3 steps as often as necessary: divide, distribute, change signes and add.

- When you get to a point where the degree you're left with is lower than the degree of the divisor, you have a remainder.

- Your answer is the quotient you have on top. If there's a remainder, it

- Your answer is the quotient you have on top. If there's a remainder, it goes over the divisor.

PRACTICE PROBLEMS: FIND EACH QUOTIENT.

1. $(2x^2-9x-5) \div (x-5)$
2. $(5x^2+2x-7) \div (x-1)$
3. $(24x^3+8x^2+6x+4) \div (4x+2)$
4. $(7x^3-17x^2+55x-21) \div (7x-3)$
5. $(14x^3-40x^2+12x+24) \div (2x-4)$
6. $(2x^3+8x^2-32x+16) \div (2x-4)$
7. $(15x^7 + 16x^6 + 4x^5 + 25x + 10) \div (5x+2)$
8. $(15x^5 + 41x^4 + 28x^3 + 15x^2 + 20x) \div (3x+4)$
9. $(x^3-4x^2+2x+5) \div (x-2)$
10. $(2x^3+4x^2-5) \div (x+3)$
11. $(12x^3-11x^2+9x+18) \div (4x+3)$
12. $(x^3-8) \div (x-2)$
13. $(x^3+2x^2-4x-6) \div (x+2)$
14. $(x^3+130) \div (x+5)$
15. $(2x^3-4x-6) \div (x-2)$
16. $(x^3 -27) \div (x-3)$
17. $(2x^3+2x^2- 4x- 5) \div (x-2)$
18. $(x^3+2x -30) \div (x+6)$
19. $(3x^3-4x^2-6) \div (x-3)$
20. $(x^3 -3x -27) \div (x+3)$

39. FACTORING USING GREATEST COMMON FACTOR

Q: What do you call friends who love math?

A: Algebros

Factoring, like solving equations, is going to be one of those very, very important topics in Algebra, one that you'll want to ensure you can do correctly every time.

To factor an expression means to break it down into 2 or more expressions that are multiplying each other. Just as with numbers, each algebraic expression has a unique set of factors when it's factored completely. There will be a number of different approaches you'll want to try when determining that unique factorization for the expression you're faced with.

The first method you'll always want to try is Greatest Common Factor —GCF. As you probably remember from elementary school, the GCF is the largest expression that all the terms share.

To factor using GCF:

- Find the GCF of the coefficients. Remember, that's the largest factor that they all share.
- For each variable, find the largest power shared by all terms.
- Your GCF is that coefficient multiplied by the largest power of each variable shared by each term.
- The GCF is written first, followed by one set of parentheses.
- To determine what goes into the parentheses, divide each term by the GCF. The quotients go into the parentheses.
- If you choose to check your answer, distribute the GCF by the terms inside the parentheses. Your answer should be the original problem.

BE CAREFUL!!! The GCF cannot have a higher exponent than the lowest of the shared exponents. So, for example, if your terms are x^2, x^5 and x^9, your GCF is x^2, since that's the most the first term can share.

BE CAREFUL!!! There are a lot of methods used for factoring a polynomial. GCF is ALWAYS the one you try FIRST.

EXAMPLE: FACTOR $5x^2-15x$

Answer: $5x(x-3)$

PRACTICE PROBLEMS: FACTOR:

1. $3x+6+ 24x$
2. $4y+28$
3. $16x-40y$
4. $34x-82$
5. $65x-39$
6. $35x^2-25x$
7. x^3-6x^2+x
8. $48x^2-54x$
9. $16x^4y^2-32x^4y^9+4xy^3$
10. $121xy-44xy^2+55x^2y$
11. $63xy- 49x$
12. X^4-7x^2+3x
13. $12x-30$
14. $48x^2+30x$
15. $144x^5+84x^4-48x$
16. $54x^8-81x^3$
17. $200x^7- 160x^4 +40x^3$
18. $5x^3-45x^2+75x$
19. $27x^4y^3-81xy$
20. $4x^6-84x^4+100x$
21. 21. $6x-18$
22. $5x^2 + 20x$
23. $8x^3+ 24x$
24. $8x^2+16x + 8$
25. $ax^2 +bx$

40. FACTORING BY GROUPING

Q: What do you call a number that can't keep still?

A: A roamin' numeral.

Sometimes you'll have to factor a polynomial that has no common factor. One approach that sometimes works if factoring by Grouping:

- Split the polynomial into two halves. Generally, the left and the right half will work out just fine.

- Find the GCF for the terms on the left and factor it.

- Find the GCF for the terms on the right half and factor it.
- Compare the two sides. If the terms inside the parentheses are the same, it becomes your new GCF.

- That GCF goes first. Into a new set of parentheses place the other terms.

BE CAREFUL!!! Sometimes the two halves have almost-the-same GCFs—that's not good enough. If the only difference in the GCFs is the signs, try factoring out a negative number or expression from one of the halves. It should change all the signs in that set of parentheses.

BE CAREFUL!!! Sometimes one of the halves will appear to have no GCF. In this case, it's permissible to factor out a 1 or a -1. This is pretty much the only time you'll want to use those particular factors.

EXAMPLE: FACTOR $x^2-x+xy-y$

ANSWER: LEFT: $x(x-1)$ RIGHT: $y(x-1)$

$(x-1)(x+y)$

Your answer can be checked by FOIL if you choose.

PRACTICE PROBLEMS: FACTOR BY GROUPING:

1. $ax + ay + bx + by$
2. $2xy + 6x + y + 3$
3. $2xy - 2x + 3y - 3$
4. $6xy - 9x + 2y - 3$
5. $2xy + 2x + 5y + 5$
6. $3xy - 12x + y - 4$
7. $6x^2 - 30xy + x + 5y$
8. $6xy + 3xb + 2ay + ab$
9. $3x^2y + 3x - xy^2 - y$
10. $2ax - 2ay + bx - by$

41. FACTORING THE DIFFERENCE OF PERFECT SQUARES

Q: Why is 6 afraid of 7?

A: Because 7 8 9

As I hope you remember from elementary school, a perfect is the product of 2 equal factors. In terms of numbers, the first 13 factors are:

1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144 and 169. (You know how I got those, right?? 1^2, 2^2, 3^3 …) You'll want to have them memorized, so that you can recognize them.

In terms of variables, a variable is a perfect square if it has an even exponent. That makes sense when you remember the rules for multiplying powers of the same base: add the exponents. If you add an exponent to itself, the resulting exponent will be even.

This type of factoring will ONLY work when it's the Difference of Perfect Squares. (You won't learn how to factor the sum of perfect squares, or the difference of number that aren't perfect squares, until Junior or Senior Year.)

So your first job is to check:

- Is it a difference?? That means it NEEDS a subtraction sign, not addition.
- Are all the coefficients and constants perfect squares?
- Are all the exponents even numbers?

If any of your answers was "No" then you need a different manner of factoring. But if you got all "Yes" answers, then the process is actually pretty simple:

- Set up two sets of parentheses.
- "Unsquare" your first term and put your answer first in each set of parentheses. If your exponent is even, simply cut it in half.
- "Unsquare" your second term and put your answer second in each set of parentheses. If your exponent is even, simply cut it in half.
- One parentheses gets a "+" sign, the other a "-"
- Your answer can be checked by FOIL if you choose.

EXAMPLE: Factor $9x^6 - 25$

Answer: $(3x^2+5)(3x^2-5)$

PRACTICE PROBLEMS: FACTOR:

1. X^2-36
2. X^2-81
3. X^2-1
4. X^2-25
5. $49-X^2$
6. $121 - X^2$
7. $X^2 - 100$
8. X^6-16
9. $144x^{10}-Y^{12}$
10. $100X^2-Y^8$
11. $64x^2y^4-49z^{12}$
12. x^4-81
13. $400 - x^6$
14. $9-y^4z^8$
15. $64X^{10}- 0.04$
16. $0.25x^{14}- y^4$
17. $169 - x^2$
18. $4-x^2$
19. $25x^2-y^4$
20. $4x^4 - y^{12}z^{20}$
21. $x^6 - 100$
22. $49x^2 - 64y^4$
23. $100 - 81x^2$
24. $121 - 4x^4$

42. FACTORING TRINOMIALS

Q: What does the zero say to the the eight?

A: Nice belt!

Take another deep breath… this is about to get a bit more complicated.

Factoring trinomials is a bit more challenging than the ways that have preceded it. (But not quite as challenging as the topic to follow it!!!)

Your first step, of course, is to make sure there's no GCF you can factor out. Let's assume you've done that.

Your next step is to make sure the leading coefficient—the coefficient of the term with the highest exponent—is 1. (If it's not, then go to the next section.) And to ensure that your trinomial is in standard form, so the exponents count down.

OK, ready to begin?

- First, set up 2 sets of parentheses

- Next, break down the first term. If it's x^2, that means each parentheses will start with an x.

- List the factors of the constant.

- From that list, determine the pair whose sum or difference will give you the middle coefficient.

- Place the numbers in that pair into the 2nd spot in each set of parentheses.

- The sign of the larger number will be the same sign as in the middle term of the trinomial

- If the trinomial's constant has a positive sign, then each set of parentheses will share the same sign as each other. If it has a negative sign, then they'll have opposite signs.

Got all that?? Honestly, if you're decent with your times tables, it's not as bad as it looks.

Let's slowly do an example. I'll set it up as a chart so you can see the reasoning.

FACTOR: $x^2 + 2x - 15$

STEP	REASONING
( ) ( )	- First, set up 2 sets of parentheses
(x)(x)	- Next, break down the first term. If it's x^2, that means each parentheses will start with an x.
1, 15 3, 5	- List the factors of the constant.
3, 5	- From that list, determine the pair whose sum or difference will give you the middle coefficient. (5-3=2)
(x 3) (x 5)	- Place the numbers in that pair into the 2nd spot in each set of parentheses.
(x 3)(x+5)	- The sign of the larger number will be the same sign as in the middle term of the trinomial (the "+2x" gives the larger number, the 5, a +)
(x-3)(x+5)	- If the trinomial's constant has a positive sign, then each set of parentheses will share the same sign as each other. If it has a negative sign, then they'll have opposite signs.

Your answer can be checked by FOIL if you choose.

PRACTICE PROBLEMS: FACTOR EACH TRINOMIAL:

1. $x^2 + 4x + 3$
2. $x^2 + 6x + 5$
3. $x^2 + 7x + 6$
4. $x^2 + x - 20$
5. $x^2 - 2x - 35$
6. $x^2 + 4x + 4$
7. $x^2 - 2x - 48$
8. $x^2 + 13x + 36$
9. $x^2 - 7x + 10$
10. $x^2 + 6x - 16$
11. $x^2 + 34x - 72$
12. $x^2 + 17x + 52$
13. $x^2 + 10x + 25$
14. $x^2 - 7x + 12$
15. $x^2 - 15x + 14$
16. $x^2 + 10x - 56$
17. $x^2 + 24x - 81$
18. $x^2 + 15x + 36$
19. $x^2 - 21x - 25$
20. $x^2 + 12x - 64$
21. $x^2 - 6x + 9$
22. $x^2 - x - 20$
23. $x^2 - 2x - 80$
24. $x^2 - 3x - 18$
25. $x^2 + 4x - 12$
26. $x^2 + 10x + 9$
27. $x^2 + 4x + 4$
28. $x^2 - 2xy + y^2$
29. $x^2 + 3xy + 2y^2$
30. $x^4 + 4x^2 + 3$
31. $x^2 - 8x + 7$
32. $x^2 - 12x + 20$
33. $x^2 + 8x + 15$
34. $x^2 - 9x + 20$
35. $x^2 + x - 56$

43. FACTORING TRINOMIALS WITH A LEADING COEFFICIENT

Q: What do you get if you cross a math teacher with a crab?

A: Snappy answers.

Sometimes trinomials have a leading coefficient (other than 1—they always have SOME coefficient) that you can't factor out. When that happens, we rely on a method called "the Eyeglass Method." (Though I must admit, it takes either a very creative mind or better drawing ability than I posses to actually see eyeglasses in this method.)

Here is the process:

- Multiply the leading coefficient by the constant. (circling these two terms form the lenses of the eyeglasses.)

- Take that product, and list all its factors.

- Choose the set of factors that add or subtract to the middle term, and determine the signs to make it work

- Substitute those two terms for the middle term. (An inverted "V" forms the nosepiece for the eyeglasses.)

- Now use factoring by grouping to factor the resulting polynomial.

- Your answer can be checked by FOIL if you choose.

Here's an example, once again in chart form so you can follow the reasoning:

FACTOR $2x^2 - 5x - 12$

STEP	REASONING
$2(-12) = -24$	- Multiply the leading coefficient by the constant
1, 24 2, 12 3, 8 4,6	- Take that product, and list all its factors.
3, -8	- Choose the set of factors that add or subtract to the middle term, and determine the signs to make it work (3 + -8 will give you the -5 in the trinomial.
$2x^2 - 8x + 3x - 12$	- Substitute those two terms for the middle term.
$2x(x-4) + 3(x-4)$ $(x-4)(2x+3)$	- Now use factoring by grouping to factor the resulting polynomial. -

PRACTICE PROBLEMS: FACTOR:

1. $2x^2-9x-5$
2. $3x^2-2x-8$
3. $4x^2+13x-3$
4. $5x^2-9x-2$
5. $6x^2-21x-12$
6. $5x^2+2x-7$
7. $4x^2+21x+27$
8. $3x^2+2x-16$
9. $2x^2+11x+15$

. 7. $4x^2+21x+27$

8. $3x^2+2x-16$

9. $2x^2+11x+15$

10. $6x^2+11x+15$

11. $12x^2+31x+20$

12. $12x^2-4x-1$

13. $10x^2+9x+2$

14. $8x^2+14x+3$

15. $6x^2+13x+6$

16. $8x^2—6x+1$

17. $3x^2+4x-7$

18. $25x^2-10x+1$

19. $12x^2+28x + 15$

20. $6x^2+11x-10$

21. $2x^2 +5x -3$

22. $6x^2 +23x -4$

23. $3x^2 -x-10$

24. $2x^2 -7x -4$

25. $2x^2 + 15x +18$

26. $2x^2 +5x -7$

27. $3x^2 +8x -3$

28. $6x^2 -5x -4$

29. $4x^2 +4x -3$

30. $6x^2 +7x -5$

44. FACTORING COMPLETELY

Q: What do you get if you add two apples and three apples?

A: A middle school math problem!

When factoring a polynomial, your ultimate goal is to factor it completely—to get to the point where none of the individual factors can themselves be factored. For some polynomials, as in the examples below, it can be more than a single step.

The order in which you should try methods is:

a) GCF
b) Difference of Squares—only good if it's a binomial
c) Trinomial—only good if it's a trinomial
d) Grouping—frequently works for polynomials of 4 terms

The directions "Factor completely" are actually kind of misleading—EVERY factoring problem should be done completely!!! But if you do see those directions, they're frequently a hint that the problem needs several steps to complete.

Some examples:

Example 1: Factor $3x^3-3x^2-60x$

Answer: $3x(x^2-x-20) = 3x(x-5)(x+4)$

Example 2: Factor: x^4-1

Answer: $(x^2+1)(x^2-1) = (x^2+1)(x+1)(x-1)$

PRACTICE PROBLEMS: FACTOR COMPLETELY:

1. $3x^2-3$
2. $2x^2-4x-6$
3. x^3-9x
4. $8x^2+8x-16$
5. x^4-81
6. $5x^2-20x+20$
7. $10x^2-90$
8. $Ax^2-Ax-56A$
9. $8x^2-50$
10. x^3-2x^2-15x
11. x^4-10x^2+9
12. $4x^2-100$
13. $6xy^2-30xy+36x$
14. ace +bce +ade +bde
15. x^3-121x
16. $3x^2-432$
17. $9x^2-36$
18. x^4-25x^2-25
19. $5x^2-10x-175$
20. X^8-1

45. SOLVING QUADRATIC EQUATIONS

Q: How do you know when you've reached your Math Professors voice-mail?

A: The message is "The number you have dialed is imaginary. Please, rotate your phone by 90 degrees and try again..."

A Quadratic Equation is a second degree equation—one which contains a variable squared. (OK, I can already hear your question: if "quad" means 4, then why is a second degree equation called quadratic? One of my students looked it up a year or two ago, here's the response she emailed me:

"Web Site: http:// mathforum. org/library/drmath/view/52572.html

Date: 05/22/99 at 20:40:32
From: Doctor Peterson
Subject: Re: Why is an equation having only two roots, one of which is raised to 2, called a "quadratic equation"?

People often wonder about the word "quadratic," because they know that "quad" usually means "four," yet quadratic equations involve the second power, not the fourth. But there's another dimension to the word.

Although in Latin the prefix "quadri" means four, the word "quadrus"means a square (because it has four sides) and "quadratus" means "squared." We get several other words from this: "quadrille," meaning
a square dance; "quadrature," meaning constructing a square of acertain area; and even "square" (through French).

Quadratic equations originally came up in connection with geometric problems involving squares, and of course the second power is also called a "square," which accounts for the name. The third-degree
equation is similarly called a "cubic," based on the shape of a third power. ..."

OK, happy?? Now back to the point:

The process involved in solving a quadratic equation is:

- Set one side equal to zero. (Hint: it's almost always easier to solve a quadratic if the leading coefficient is positive. So keep that in mind as you determine which side will equal zero.)
- Factor the other side completely.
- "T off"—set each of the factors equal to zero and solve.

And now, of course, you can see why we spent so much time on factoring!

Example: Solve $x^2 + 4x = -3$

Answer: $x^2 + 4x + 3 = 0$

$(x+3)(x+1) = 0$

$x+3 = 0, x+1 = 0$

$x = -3, x = -1$ $\{-3, -1\}$

Example: Solve: $x^2 = 3x$

Answer: $x^2 - 3x = 0$

$\underline{x \quad (x-3) = 0}$

$x=0, \quad x-3 = 0$

$x=0, \quad x=3 \quad \{0,3\}$

PRACTICE PROBLEMS: SOLVE:

1. $4x^2=25$
2. $x^2=3x$
3. $x^2-2x=24$
4. $x^2-20=x$
5. $x^2=4x$
6. $x^2-3x=10$
7. $x^2-7= -6x$
8. $x^2-3x=70$
9. $x(x-5)=14$
10. $x(x+2)+1=49$
11. $x^2-9x=22$
12. $3x^2-x=4$
13. $2x^2+10= -9x$
14. $x(2x+4)=3-x$
15. $x^2-12x=13$
16. $x(2x-4)=x(3x+2)$
17. $x^2-15x= -56$
18. $x^2-169 =26x$
19. $x^2-72 =21x$
20. $x^2-15x =54$
21. $x^2 -5x=24$
22. $x^2 -20 = x$
23. $x^2 - 12x = 11$
24. $x^2 =28x$
25. $4x^2 = 100$
26. $x^2 =9x$

46. RATIONAL EXPRESSIONS

Q: What did one math book say to the other?

A: Don't bother me I've got my own problems!

Remember back when we talked about sets of numbers, we said that a Rational Number is any number that can be expressed as a fraction. Well, a rational expression is an Algebraic fraction. And we can perform the same operations with rational expressions as we can with numeric fractions.

OK, now let's go back to a concept you learned in elementary school. You learned that it's impossible to divide by zero, and that doing so was "undefined." Let's put that in terms of rational expressions.

Any rational expression is undefined when its denominator equals zero. So the first step in many problems will be finding those values that make it undefined. All you need to do is set the denominator equal to zero, and then solve the resulting equation. Whatever value or values you get are the one(s) that make the expression undefined.

EXAMPLE: for what values is the expression undefined?

Answer: Set the denominator, x^2-9, equal to zero. That gives you a quadratic that you can factor and solve: $(x+3)(x-3)=0$. $x=3$, $x=-3$. So the values $x=3$ and $x=-3$ are the ones that make the expression undefined

PRACTICE PROBLEMS: Determine the values for which each expression is undefined.

1. $3/x$
2. $5/(x+3)$
3. $(x-2)/(x+6)$
4. $(x-9)/(x+9)$
5. $2x/(x+6)$
6. $(x+12)/(x-3)$
7. $(x+4)/x^2$
8. $(x-5)/(x^2-4)$
9. $x/(x^2-x-20)$
10. $(3x-7)/(3x^2-25)$
11. $(x-7)/(x^2+2x-24)$
12. $(x^2-3)/(x^2-15x+56)$
13. $(x-8)/(x^2-4x)$
14. $(2x+3)/(x^3-x)$
15. $(x-16)/(x^2-100)$

47. SIMPLIFYING RATIONAL EXPRESSIONS

Teacher: Why are you doing your multiplication on the floor?

Student: You told me not to use tables.

In elementary school, you learned how to reduce fractions to lowest terms: Cancel out any common factors until there are none left to cancel. That's pretty much the same process we'll use to simplify rational expressions. Except, of course, that we'll have to factor both the numerator and denominator of the fraction so we can find those common factors.

Here's the process:

- Factor both the numerator and denominator completely
- Cancel out any common factors

BE CAREFUL!!! When two factors are opposites, (like 6 and -6 for example,)they cancel and give you a quotient of -1. Opposite factors have the exact same terms, except that all the signs have been changed. One example of opposites would be (x-2) and (-x+2)

EXAMPLE: Simplify: $\frac{x^2-3x+2}{x^2-4}$

Answer: $\frac{(x-2)(x-1)}{(x-2)(x+2)}$

$\frac{x-1}{x+2}$

PRACTICE PROBLEMS: SIMPLIFY:

1. $(x^2-4)/(x^2+3x+2)$
2. $(x^2-6x)/(x^2-36)$
3. $(x^2+4x)/(x^2-16)$
4. $(x^2-x-20)/(x^2+3x+2)$
5. $(x^2-x-6)/(x^2+3x+2)$
6. $(x^2+10x)/(x^2-4x)$
7. $(3x^2-27)/(x^2-2x-10)$
8. $(x^2+6x+5)/(x^2-25)$
9. $(x^2-x-42)/(x^2+13x+42)$
10. $(x^2+4x)/(x^2+7x+12)$
11. $(x^2+2x-24)/(x^2-10x+24)$
12. $(x^2+3x-18)/(x^2-9x-18)$
13. $(x^2-25)/(x^2-10x+25)$
14. $(x^2+x-20)/(x^2-9x+20)$
15. $(x^2+12x+36)/(x^2-11x+30)$

16. $(x^2-5x+6) / (x^2+2x-8)$

17. $(x^2+10x+24) / (x^2-2x+24)$

18. $(x^2+9x+18) / (x^2-3x-18)$

19. $(x^2-2x-24) / (x^2-36)$

48. MULTIPLYING RATIONAL EXPRESSIONS

There are three people applying for the same job. One is a mathematician, one a statistician, and one an accountant.

The interviewing committee first calls in the mathematician. They say "we have only one question. What is 500 plus 500?" The mathematician, without hesitation, says "1000." The committee sends him out and calls in the statistician.

When the statistician comes in, they ask the same question. The statistician ponders the question for a moment, and then answers "1000... I'm 95% confident." He is then also thanked for his time and sent on his way.

When the accountant enters the room, he is asked the same question: "what is 500 plus 500?" The accountant replies, "what would you like it to be?"

They hire the accountant.

When you multiply two fractions, the procedure is fairly simply. Cancel out any common factors, then multiply what's left, right?

The process for multiplying algebraic fractions is pretty much the same:

- Factor each numerator and denominator completely.
- Cancel any common factors.
- The answer is the product of any remaining factors.

But this bears repeating:

BE CAREFUL!!! When two factors are opposites, (like 6 and -6 for example,)they cancel and give you a quotient of -1. Opposite factors have the exact same terms, except that all the signs have been changed. One example of opposites would be (x-2) and (-x+2)

EXAMPLE: Find the product: $\dfrac{x^2+4x+3}{x^2+3x} \bullet \dfrac{x^3-x^2}{x^2-1}$

Answer: $\dfrac{(x+3)(x+1)}{x(x+3)} \bullet \dfrac{x^2(x-1)}{(x+1)(x-1)}$

$\dfrac{x}{1}$

PRACTICE PROBLEMS:

1. $\frac{x^2+5x+6}{x^2-4} \cdot \frac{x^2-2x}{x^2-9}$

2. $\frac{x^2+4x}{x^2-16} \cdot \frac{6x-24}{3x^2}$

49. DIVIDING RATIONAL EXPRESSIONS

Q. What shape is usually waiting for you at Starbucks?

A. A line.

To find the quotient of fractions, you learned in elementary school to invert the expression that comes after the division sign (the divisor) and multiply the results. (You may remember an expression like "keep, switch, flip" or something similar.)

The process for dividing algebraic fractions is almost the same:

- Factor all numerators and denominators
- Invert (flip) any fraction following a division sign
- Cancel any factor on top with its match on the bottom, whether or not the factors are in the same fraction.
- And remember, once again, that opposite factors will yield a quotient of -1.

EXAMPLE: DIVIDE: $\dfrac{x^2 - 16}{9x^2-25} \div \dfrac{x-4}{3x+5}$

$$\frac{(x-4)(x+4)}{(3x-5)(3x+5)} \cdot \frac{(3x+5)}{(x-4)}$$

$$\frac{x+4}{3x-5}$$

50. FINDING A COMMON DENOMINATOR

Q: Who invented algebra?

A: A Clever X-pert.

As you know, in order to add or subtract fractions, they need to share a common denominator. If they don't already have that common denominator, you'll need to follow these steps to find the LCD:

- Factor each of the denominators
- The LCD is the product of each of the factors that appear in any of the denominators, to the highest power to which they appear.

So, for example, is one denominator had an x^2 and the other had an x^3, the LCD would be x^3.

Once you've found the LCD, you can convert each fraction to an equivalent fraction containing that LCD. Just multiply each numerator by whichever factors are NOW in the denominator that weren't originally there. From that point, you can add or subtract the fractions.

$$\frac{4x^2}{x(x-2)(x+2)} + \frac{2x^2+4x}{x(x-2)(x+2)}$$

EXAMPLE: Convert to fractions with a common denominator:

$$\frac{4x}{x^2-4} + \frac{2x}{x^2-2x}$$

$$\frac{4x}{(x-2)(x+2)} + \frac{2x}{x(x-2)}$$

$$\frac{4x^2}{x(x-2)(x+2)} + \frac{2x(x+2)}{x(x-2)(x+2)}$$

$$\frac{4x^2}{x(x-2)(x+2)} + \frac{2x^2+4x}{x(x-2)(x+2)}$$

$$\frac{4x^2+2x^2+4x}{x(x-2)(x+2)}$$

$$\frac{6x^2+4x}{x(x-2)(x+2)}$$

51. ADDING AND SUBTRACTING FRACTIONS

Q: What is a smart bird favorite type of math?

A: owl-gebra

Once you've found the LCD, it's fairly easy to add or subtract algebraic fractions. Simply add or subtract the numerators, then reduce your fraction to simplest form.

BE CAREFUL!!!! When subtracting, don't forget to distribute the negative sign to the numerator after the subtraction sign.

BE CAREFUL!!!! Remember, all algebraic fractions should be put into simplest form. That means that after you're done adding or subtracting, you'll need to factor both the numerator and the denominator in order to cancel out any common factors. Keep in mind, too, that additive inverses cancel and give a quotient of -1.

EXAMPLE: SUBTRACT -

$$\frac{6}{2(y+4)} - \frac{8}{y+4}$$

$$\frac{6}{2(y+4)} - \frac{8(2)}{2(y+4)}$$

$$\frac{6-16}{2(y+4)}$$

$$\frac{-10}{2(y+4)}$$

$$\frac{-5}{y+4}$$

52. EQUATIONS WITH FRACTIONAL COEFFICIENTS

Teacher: "What is seven Q plus three Q?"

Student: "Ten Q"

Teacher: "You're Welcome."

Sometimes it becomes necessary to use a fractional coefficient in a problem—to deal with half an x or a third of a y. When that happens, there's a process designed to make things a bit easier for all those fraction haters out there.

Come on, admit it: you hate fractions, right? The easiest way to deal with fractional coefficients is to simply get rid of them: multiply each term in the equation by the LCD. What that accomplishes is it cancels out all the denominators in the problem, leaving you with a nice, easy equation to solve.

EXAMPLE: $\frac{1}{2}x - x = 8$

ANSWER: the LCD is 6, so multiply each term by 6, and cancel:

$$6(\tfrac{1}{2}x) - 6(x) = 6(8)$$

$$3x - 2x = 8$$

$$X = 8$$

BE CAREFUL!!!! Don't forget to multiply EACH TERM—including the other side of the equation—by the LCD!!!

Practice Problems

53. EQUATIONS WITH DECIMAL COEFFICIENTS

Surgeon: Nurse! I have so many patients! Who do I work on first?

Nurse: Simple. Use the order of operations.

As with fractional coefficients, the easiest way to deal with decimal coefficients is to get rid of them. Simply move the decimal point of each coefficient enough places that there are no more decimal coefficients.

BE CAREFUL!!! Remember, what you do to one term, you must do to ALL terms in the equation. So if you move one decimal three places to change a 0.125x into a 125x, you'll have to move the decimal point for all the other terms 3 places to the right as well. (What you've done, of course, is multiply each term by 1000.)

EXAMPLE: =0.5X + 12 = 0.3X -8

ANSWER: First move all the decimal points 1 place to the right to clear out the decimals:

5x + 120 = 3x -80. Now solve:

2x + 120 = -80

2x = -200

X = - 100

PRACTICE PROBLEMS:

1. 0.5X +20 = 0.7X
2. 2.2X +44= 3.3X
3. 0.05X + 36 =X
4. 12.5X - 6 = 10X
5. 1.6X + 100 =2X
6. 2.3X - 5.4 = 2X +1.8
7. 0.03X + 100 = 0.05X
8. 6.4X -24 = 5.2X
9. 3X +2.5X =44
10. 0.2X + 120 =0.5X
11. 6X-24 =4.5X
12. 0.2(X+40) = 0.4X
13. 0.5(2X+12) =2X
14. 1.25X + 48 = 2X
15. 6X + 12.4 =8X
16. 4.8X - 16 = 3.2X

54. VERBAL PROBLEMS: INVESTMENT

Knock, Knock.

Who's there?

Polly.

Polly who?

Polynomial. Why the third degree?

Every time you put a birthday check into your bank account, you're investing money. The idea is that you lend some of your money to a financial institution—in this case a bank, though the same idea works with stocks. In return for your placing your money in one of their accounts, they pay back what you gave them, as well as giving you extra money. That extra money is called Interest, and the money you invested is known as the Principal. The interest is a percent of the principal, expressed as a decimal. (So an interest rate of 4% would be expressed as 0.04.

BE CAREFUL!!!! Remember what you learned in elementary school about converting percents to decimals. You move the decimal two places to the left, adding a zero if necessary.

The formula you'll be using is A= PRT, where P= the principal, r= the interest rate expressed as a decimal, and t is the time in years. We use A (for "amount") to signify the amount of money earned (or "income"—but we don't want two "I" words in the same formula.) Like so many of the other verbal problems we've covered, these involve a chart, and we'll multiply the first 3 columns to get the last one.

Principal	Rate	Time	Amount

BE CAREFUL!!! Many of the problems we'll be doing will involve either the word "annual" or the word "yearly" when referring to the amount of time. Either one will imply that the amount of time is 1. That won't always be the case, particularly when you see these problems again in Algebra II

EXAMPLE: Mrs. Smith invested $7500 in 2 parts: one part at 6% and the other at 10%. Find the amount invested at each rate if the total yearly income is $590.

ANSWER: Let x = the amount at 6%.

Let 7500 – x = the amount at 10%

P	R	T	A
X	0.06	1	0.06X
7500 -X	0.10	1	0.10(7500 –X)

If the total income is $590, that's the sum of the last column:

0.06x- 0.10(7500 –x) = 590.

What many people find easiest is to clear out the decimals at the beginning—multiply each term by 100 and move all the decimal points 2 places over.

6X – 10(7500 –X) =59,000

6X – 75,000 -10X = 59,000

-4X – 75,000 = 59,000

-4X = -16,000

X = 4000.

She invested $4000 at 6% and the remaining $3500 at 10%.

PRACTICE PROBLEMS:

1. A total of $8,000 is deposited into 2 savings accounts. The interest is 12% for one account ant 12% for the other. The total annual interest was $900. How much was invested in each account?

2. A total of $7,000 was invested in two accounts such that the annual interest earned in each account was the same. The interest rates of the two accounts were 15% and 20% respectively. Find the amount invested at each rate.

3. Time invested some money into a 10% account. In a 16% account, he invested $4000 more than the amount in the 10% account. He earned a total of $2200 annual interest on these investments. How much was deposited into each account?

4. A man invested some money at 8% and twice as much money at 10%. His total annual income was $420. Find the amount invested at each rate.

5. Mrs Kim invested some money at 5% and twice as much at 7%. Her total annual income was $420. Find the amount invested at each rate.

6. A total of $2,200 is invested, part at 5% and the rest at 3%. The annual income from the 3% investment is $46 less than the income from the 5% investment. Find the amount invested at each amount.

7. George invested $19,000 for one year, part at 11% and the rest at 12%. The total interest he earned was $2,200. How much did he invest at each rate?

8. Cindy invested $24,000 for one year, part at 6% and the rest at 9%. The total interest she received was $1,740. Find the amount invested at each rate.

9. You inherit $16,000 and invest it in two accounts, one paying 6% and the other 8% annual interest. At the end of one year, the interest from the 8% account is $580 more than the interest from the 6% account. Find the amount invested at each rate.

55. VERBAL PROBLEMS: MIXTURE

Q: How is the moon like a dollar?

A: They both have 4 quarters.

Here's the scenario: you and a bunch of your friends go to Roosevelt Field, to Dylans' Candy Bar (Is it still there?? If not, then play along anyway, OK?) You each choose a type of candy at a different price, and decide how much of it you want. Then, you all combine your bags, and split the candy and the price evenly.

Hungry yet??

This is the basic idea behind mixture problems: taking several different items and mixing them (hence the name) so that the price falls somewhere between the highest and lowest prices.

Like so many other verbals, these will include a chart. (And, like ALL other verbals, these will of course include LET statements.)

Think for a second about the columns, and the information included in a problem like this one. You'll need to classify the type of candy. You'll need to know the number of pounds (or in some cases, the number of ounces or whatever), and you'll need to know the price per pound or per ounce. Once you have that, you can multiply the amount by the price to find a total:

Type	# of pounds	Price per pound	Total

BE CAREFUL!!! Mixture problems are different from the other types of verbal problems we've done because they include a row for the mixture. The sum of the totals of the types is the mixture, as shown below.

EXAMPLE: Tommy mixed 3 pounds of Tootsie Rolls, at \$3 per pound, with some Snickers, at \$3.60 per pound. He ended up with a mixture that sold at \$3.40 per pound. How many pounds of Snickers did he mix?

Answer: Let x = the number of pounds of Snickers

Type	# of pounds	Price per pound	total
Tootsie Rolls	3	3	3(3)
Snickers	x	3.60	3.60x
Mixture	X+3	3.40	3.40(x+3)

$3(3) + 3.60\,x = 3.40\,(x+3)$

$9 + 3.60x = 3.40(+3)$

$90 + 36x = 34(x+3)$

$90 + 36x = 34\,x + 102$

$90 + 2x = 102$

$2x = 12$

$X=6$

Tommy mixed 6 pounds of Snickers.

Note that when I moved the decimal, I only moved it one place. Sure, I could have moved it 2, but one was all that was necessary to clear out the decimals.

PRACTICE PROBLEMS:

1. Two pounds of organic tea that costs $6.75 per pound is mixed with some generic tea that costs $3.75 per pound. How many pounds of the generic tea should be used to make a new tea mixture that costs $4.65 per pound?

2. A special treat is made by mixing 5 pounds of popcorn that costs $0.80 per pound with caramel costing $2.40 per pound. How many pound of caramel are needed to make a mixture that costs $1.40 per pound?

3. Teri wants to mix some blueberries that cost $2.75 per pound with 2 pounds of raspberries that cost $3.50 per pound to get a mixture that costs $3.25 per pound. How many pounds of blueberries should Teri include?

4. Tim wants to mix peanuts costing $1.90 per pound with raisins costing $1.20 per pound to make a 10 pound mixture costing $1.48 per pound. How many pounds of peanuts should Tim include?

5. Find the selling price per quart of a mixture made from 13 quarts of a punch costing $0.80 per quart with 7 quarts of soda costing $1.20 per quart.

56. VERBAL PROBLEMS: PERCENT-MIXTURE

Q: How can you add eight 8's to get the number 1,000? (only using addition)

A: 888 +88 +8 +8 +8 = 1,000

Percent mixture problems tend to read a lot like Chemistry problems. You mix a number of ounces of one solution (which is a particular percent strong) with another, to arrive at a mixture which has a different strength. But once you get past the actual words, these are a lot like typical mixture problems.

And, like typical mixture problems, percent mixture problems involve a final row that contain information about the mixture. Once again, the bottom entry in the last column is the sum of the ones above it.

The chart you'll be using is:

Type	# of ounces	% iodine (or whatever)	Total

BE CAREFUL!!! Adding a substance which is "pure" means that its strength is 100%. Adding water, on the other hand, means you're adding something with a strength of 0%. (Think about it for a minute—do you really expect to find iodine or whatever in the water that comes out of your tap?? Of course not!!!)

EXAMPLE: A chemist has one solution that is 30% pure acid and one that is 60% pure acid. How many pounds of each solution must be used to produce 60 pounds of a mixture that is 50% pure acid?

ANSWER: LET x = pounds of 30% solution

LET 60-x = pounds of 60% solution (because the total of the 2 is 60 pounds)

Type	# pounds	% pure acid	Total
1st	X	0.30	0.30X
2nd	60-X	0.60	0.60(60-X)
Mixture	60	0.50	0.50(60)

$0.30X + 0.60(60 - X) = 0.50(60)$

$3X + 6(60 - X) = 5(60)$

$3X + 360 - 6X = 300$

$-3X + 360 = 300$

$-3X = -60$

$X = 20$

The chemist needs 20 pounds of the 60% pure acid solution.

PRACTICE PROBLEMS:

1. How much pure acid must be added to 15 grams of an acid solution that is 40% acid in order to produce a solution that is 50% acid?

2. How many liters of a 70% alcohol solution must be added to 50 liters of a 40% alcohol solution to produce a 50% alcohol solution?

3. How many ounces of pure water must be added to 50 ounces of a 15% saline solution to make a saline solution that is 10% salt?

4. Two hundred liters of a punch that contains 35% fruit juice is mixed with 300 liters of another punch. The resulting fruit punch is 20% fruit juice. Find the percent of fruit juice in the punch that was added.

5. Ten grams of sugar are added to a 40 gram serving of a breakfast cereal that's 30% sugar. What is the percent concentration of sugar in the resulting mixture?

6. How many pounds of dog food that's 50% rice must be mixed with 400 pounds of dog food that is 80% rice to make a dog food that is 75% rice?

7. Ten quarts of pure apple juice are added to 90 quarts of a fruit juice that is 10% pure apple juice. What is the percent concentration of pure apple juice in the resulting mixture?

8. Five pounds of candy that is 20% chocolate are combined with a candy that is 40% chocolate. How many pounds of the second candy should be used to get a candy that is 25% chocolate?

9. How much water should be added to 30 cups of juice drink that is 70% pure juice to get a diluted mixture that is only 60% juice?

10. A chemist has a 21% salt solution and a 15% salt solution. How many quarts of each should be mixed to produce 60 quarts of a 19% salt solution?

57. EQUATIONS WITH RATIONAL EXPRESSIONS

Q: How many eggs can you put in an empty basket?

A: Only one, after that the basket is not empty.

Sometimes, instead (or in addition to) just having fractional coefficients, we see equations with algebraic fractions. This means that sometimes the denominator will contain a variable or several variables. That does NOT mean there's anything to worry about; our approach is the same. We multiply each term by the LCD.

BUT.. there is one very important difference between these equations and ones with rational coefficients. These ones MUST BE CHECKED. As with the Absolute Value Equations we did first trimester, these equations bring the possibility of Extraneaous Roots (answers that don't check and, as a result, aren't real answers.)

BE CAREFUL!!! Equations with rational expressions MUST be checked for extraneous roots!!!

EXAMPLE: 1/3 + 1/x = 1/12

12x(1/3) + 12x(1/x) = 12x(1/12)

4x + 12 =x

12 = -3x

-4 = x

CHECK: 1/3 + 1/(-4) = 1/12

4/12 + (-3/12) = 1/12

1/12 = 1/12

PRACTICE PROBLEMS:

1. 2x/3 + 20 = x
2. x/8 - 48 = x/10
3. 5x/6 + 16 =x
4. 160 - x/6 = x/5
5. 5/x +4 = 11
6. 2/5 - 1/x = 1/2
7. 5/8 + 2/x = 2/3
8. 6/(7x) + 12 = 24
9. 3/x - 4 =11/x
10. 2/ (3x) + 24 = 1/(2x)
11. x/4 + 15 = 20
12. 1/x - 10 = 12
13. 5/(6x) - 2 = 2/(3x)

58. VERBAL PROBLEMS WITH RATIONAL EXPRESSIONS: NUMBER PROBLEMS

Q: Where can you buy a ruler that is 3 feet long?

A: At a yard sale

This set of problems, as you've probably guessed, will involve solving equations with rational expressions. When you check your answer for extraneous roots, be sure that the answer makes sense not only in the equation, but in the actual problem itself.

EXAMPLE: The denominator of a fraction is three more than the numerator. If one is subtracted from the numerator and 4 is added to the denominator, the new value of the fraction is ½. Find the original fraction.

LET x= original numerator

LET x+3 = original denominator

LET x/(x+3) = the original fraction.

$$\frac{x-1}{x+3+4} = \frac{1}{2}$$

$$\frac{x-1}{x+7} = \frac{1}{2}$$

$$2(x+7)\frac{x-1}{x+7} = \frac{1}{2}(2)(x+7)$$

$$2(x-1) = x+7$$

$$2x-2 = x+7$$

$$X - 2 = 7$$

$$X = 9$$

The original fraction was 9/12.

PRACTICE PROBLEMS:

1. When one half of a number is increased by one third of that number, the result is 25. Find the number.
2. If 5 is added to one half of a number, the result is the same as when three-fifths of that number is decreased by 3. Find the number.
3. The larger of two numbers is 4 more than twice the smaller. If hte larger is divided by the smaller, the quotient is 3 and the remainder is 1. Find the numbers.
4. The numerator of a fraction is 8 less than the denominator. The value of a fraction is 3/5. Find the fraction.
5. What number must be added to both the numerator and denominator of the fraction 7/19 to make the value of the resulting fraction 3/4?
6. Ten less than one half of a number is is one third of the number. Find the number.
7. Two thirds of a number, decreased by 7, is one half the number. Find the number.
8.

59. VERBAL PROBLEMS: WORK

Q: When things go wrong, what can you always count on?

A: Your fingers.

Let's say you've decided to do a big job over Summer vacation—maybe to paint the garage. And you've somehow determined that, if you were to do that job alone, it would take you 30 hours.

But let's say your twin brother, who works at the exact same rate, joins you. Now how long does it take for the two of you? 15 hours, right?

And let's say that dad gets into the act. And that he can work twice as fast as you and your twin. So now how long does it take? Half as long, right? So we're down to 7.5 hours.

This is the basic idea behind work problems: several people working together to accomplish a job.

The basic equation is: (Rate of work)(time work) = part of job done.

The other thing that's different about Work problems is that the different parts of the job will add up to 1, because 1 represents 1 whole job.

BE CAREFUL!!! To determine a person's rate of work, put 1 over the length of time it would take him/her to complete the job alone. In the example above, it would take you 30 hours alone. So your rate of work—the portion of the job you could do in one hour—is 1/30 of the whole job.

EXAMPLE: Bob can dig a ditch in 6 hours. (Why he's digging a ditch, I'm not quite sure.) Randy can dig the same ditch in 3 hours. How long will it take for them to dig it together?

ANSWER: Let x = the number of hours spent working together.

Perso n	rate	# of hours	Part of job done
Bob	1/6	x	x/6
Randy	1/3	x	x/3

$x/6 + x/3 = 1$

$6(x/6) + 6(x/3) = 1(6)$

$x + 2x = 6$

$3x = 6$

$X = 2$

Working together, it will take 2 hours to dig the ditch.

Practice Problems

60. RATIO

Q: What coin doubles in value when half is deducted?

A: A half dollar.

A ratio is simply a comparison between 2 or more things. When we say that last winter was twice as snowy as the one before that, we're saying that the ratio of snow is 2:1. It's important with ratios to keep the numbers in the same order as the words.

Algebraically, ratio problems frequently involve multiplying "x" by each number in the ratio.

EXAMPLE: The angles of a triangle are in the ratio 2:3:4. Find all three angles and classify the triangle.

ANSWER: Let 2x= 1st angle

Let 3x = 2nd angle

Let 4x = 3rd angle.

2x+3x+4x = 180 (because that's the sum of the angles of a triangle)

9x = 180

X = 20

2x = 40, 3x = 60, 4x=80.

The angles measure 40 degrees, 60 degrees and 80 degrees. The triangle is acute and scalene.

PRACTICE PROBLEMS:

1. Two numbers are in the ratio 4:7. Their sum is 66. Find the numbers.
2. Two numbers are in the ratio 5:3. Their difference is 24. Find them.
3. Two numbers are in the ratio 7:1. Their sum is 84. Find them.
4. The angles of a triangle are in the ratio 2:3:5. Find them and classify the triangle.
5. The angles of a triangle are in the ratio 1:1:2. Find them and classify the triangle.
6. The angles of a quadrilateral are in the ratio 2:3:4:5. Find them.
7. The ratio of two numbers is 7:5. The larger number is 35. Find the smaller.
8. The ratio of three numbers is 5:4:3. The larger minus the smaller is 20. Find all three numbers.
9. Two consecutive even integers are in the ratio 2:1. Find both numbers.
10. Two numbers are in the ratio 10:9. Their difference is 5. find them.
11. Two numbers are in the ratio 7:3. Their difference is 36. Find them.
12. Two numbers are in the ratio 4:3. Their sum is 84. Find them.
13. Two numbers are in the ratio 6:5. Their sum is 88. Find them.
14. Two numbers are in the ratio 7:4. Their difference is 312. Find the numbers.
15. Two numbers are in the ratio 11:9. Their difference is 56. Find the numbers.
16. Two numbers are in the ratio 7:2. Their difference is 55. Find the numbers.
17. Two numbers are in the ratio 9:5. Their difference is 16. Find them.

61. PROPORTIONS

Q: Why is the longest human nose on record only 11 inches long?

A: Otherwise it would be a foot.

A proportion is defined as an equation involving ratios; basically it's two fractions set equal to each other.

In setting up a proportion, it's important to keep your terms straight. For example, if the first fraction has the price paid over the number of ounces, then the second fraction must do the same.

BE CAREFUL!!! In setting up a proportion, be sure to be consistent. Either both numerators must contain the same units, or the elements of a single fraction must. You can't mix and match.

A proportion can be written one of two ways:

- As a fraction: $\frac{A}{B} = \frac{C}{D}$ Six

- Or horizontally: A:B = C:D

In either case, the terms in the "A" and "D" postions are known as the Extremes. The terms in the "B" and "C" positions are known as the Means.

And how do you solve a proportion? Well, you could think of it as a fractional equation and multiply each numerator by the LCD. But most student prefer the tried and true method of Cross Multiplication… just set AD = BC and solve.

Of course, now that you're in Algebra, you don't have to phrase it as "Cross Multiply." Instead, you can phrase it this way: "In a proportion, the product of the means equals the product of the extremes."

But we know that you'll think of it as cross-multiplication .

EXAMPLE: The Smiths are planning a trip to Europe. If one dollar is worth 0.74 Euros, then how many Euros can they get for $500 at the same rate?

ANSWER: $\frac{1}{0.74} = \frac{500}{x}$

Note that this particular proportion places dollars over Euros both times. It's not the only possible arrangement for this proportion.

Cross multiply: $1x = 500(0.74)$

$X = 370.$

The Smiths can purchase 370 Euros with their $500.

PRACTICE PROBLEMS:

1. Six boxes of pasta cost $7.50. At the same rate, how much would 4 boxes cost?
2. Kevin can read 45 pages per hour. At the same rate, how many pages can he read in an hour and a half?
3. Tyler can run two miles in 18 minutes. At the same rate, how long would it take him to run 6.5 miles?
4. If 6 gallons of gas cost $21. At the same rate, how much would 18 gallons cost?
5. Deena gets paid $50 for baby sitting for four hours. At the same rate, how much would she earn for babysitting 8 hours?
6. A dozen roses cost $42. How much would it cost for 18 roses?
7. Ten cans of dog food cost $2.50 At the same price, how much would 17 cans cost?
8. Kelly has been been saving spare change and has saved $10 in 4 weeks. At the same rate, how long would it take to save $25?
9. John drove an 400 miles in 6 hours. At the same rate, how many additional miles can he drive in the next two hours?
10.

62. VARIATION

Why did the girl wear glasses during math class?

(Because it improves di-vison!)

The word “variation” means exactly what you think it means: to vary or to change. We’re going to be talking about two different kinds of variation: Direct and Inverse.

DIRECT VARIATION means that both quantities do the same thing. If one increases, the other does too. Direct variation problems are proportions.

EXAMPLE: x varies directly as y. If x=2 when y= 12, then find y when x = 5.

ANSWER: Set up a proportion based on that first sentence: “x varies directly as y.” That means put x over y in both ratios: 2/12 = 5/x and cross multiply:

5(12) = 2x

60 = 2x

30 =x

INVERSE VARIATION, since it IS the “inverse”, does the opposite. Instead of setting up a proportion, you set up a product. With Inverse Variation, the two terms have the same product. As one increases, the other decreases.

EXAMPLE: x varies inversely as y^2. If x=3 when y=10, then find x when y = 5..

ANSWER: set up the product based on the first sentence: $x(y^2)$

$3(10^2) = x(5^2)$

3(100) = x(25)

300 = 25x

12 =x

PRACTICE PROBLEMS:

1. x varies directly as y, and x=3 when y=5. Find x when y=20.
2. x varies directly as y, and x=4 when y=7. Find y when x =24.
3. x varies directly as y, and x=2 when y=9. Find x when y=63.
4. x varies directly as y, and x=1 when y=6. Find y when x=12.
5. x varies directly as y, and x=5 when y=12. Find x when y=72.
6. x varies directly as y^2, and x=1 when y=2. Find x when y=6.
7. x varies directly as y^2, and x=3 when y=4. Find y when x=6.
8. x varies directly as y^2, and x=2 when y=6. Find y when x=3.
9. x varies inversely as y, and x=2 when y=6. Find x when y=4.
10. x varies inversely as y, and x=8 when y=6. Find x when y=12
11. x varies inversely as y, and x=12 when y=6. Find y when x=8.
12. x varies inversely as y, and x=5 when y=8. Find x when y=4.
13. x varies inversely as y, and x=4 when y=25. Find x when y=20.
14. x varies inversely as y^2, and x=2 when y=6. Find x when y=4.
15. x varies inversely as y^2, and x=8 when y=6. Find x when y=12.
16. x varies inversely as y^2, and x=3 when y=10. Find x when y=5.

18. x varies inversely as y3, and x=1 when y=2. Find x when y=8.

19. x varies inversely as y, and x=9 when y =20. Find x when y=5.

20. x varies inversely as y, and x=15 when y=6. Find x when y=10.

21. x varies inversely as y^2, and x=2 when y=16. Find x when y=4.

22. x varies inversely as y^3, and x=1 when y=2. Find x when y=8.

23. x varies inversely as y, and x=9 when y =20. Find x when y=5.

24. x varies inversely as y, and x=15 when y=6. Find x when y=10.

63. GRAPHING IN A PLANE

What was T. rex's favorite number?

(Eight!)

Once upon a time, before there was Google or cell phones or GPS, there was something called a phone book. It was delivered once a year, and it contained the name, address and phone number of just about everyone it your town.

And it even had a map in the back! A map was something we used in the olden days to find places. Here's how it worked: It was set up as a big square. Along the bottom edge, there would be the letters from A to Z, evenly spaced out. Going up the side were the numbers from 1 to 20, again evenly spaced. And onto that grid was placed a picture of your town. If you wanted to find a Glenn Curtiss Blvd, you would check the index, which told you to look at C4. You put one finger on C, the other on 4, and moved them until they met. Viola!!! There you were, on Glenn Curtiss Blvd!!

The Phone Book company didn't invent maps. (Once upon a time, you could also get one for free at a gas station, while the attendant pumped your gas for you. But I digress.) Maps are actually based on the math of the Cartesian Plane.

The Cartesian Plane (or Rectangular Plane, or Coordinate Plane) is based on the work by French Mathematician and philosopher Rene DesCartes. It's the intersection of two perpendicular number lines. They intersect at a point called the Origin. The horizontal number line is called the x-axis, and the vertical number line is called the y-axis. Each point on the graph can be located in terms of a pair of coordinates: (x, y) . That pair of numbers, where the number matters, is called an ordered pair. The x and y axes (that's the plural of "axis") cut the coordinate plane into four quadrants, or quarters. The quadrants are numbered counterclockwise, starting with the top right. When we choose to label the quadrants—which is actually kind of rare—we use Roman Numerals: I, II, III and IV.

To plot any point on the Coordinate Plane, simply go over the number of boxes to correspond to the x-axis, and up or down in accordance with the y.

PRACTICE PROBLEMS: Plot, then identify the quadrant containing each point where possible:

1. (3,5)
2. (3,8)
3. (-1,5)
4. (5. 0)
5. (7, -8)
6. (4. -7)
7. (-4, -9)
8. (2,-7)
9. (-9,0)
10. (11, -3)
11. (-5, -16)
12. (20, -17)
13. (-11, 11)
14. (0,-7)
15. (13, -17)
16. (20,0)
17. (0, -9)
18. (1,10)
19. (4,4)
20. (-8.-8)

64. LINES AND LINEAR FUNCTIONS

What snakes are good at doing sums?

(Adders!)

We've spent a lot of time this year studying equations. What we're about to see is the relationship between lines and graphing, specifically those equations that can form straight lines.

Let's start with basics. Any line whose equation is y= a number forms a horizontal line at that number. So y=3 is a horizontal line that hits 3 on the y-axis. And y=-7 is a horizontal line that hits -7 on the y-axis. And y= 12 is a horizontal line that hits 12 on the y-axis. And y=0 is a horizontal line that hits 0… it's the x-axis!

Likewise, any equation whose equation is x= a number forms a vertical line at that number. So x=-4 is a vertical line that passes through -4 on the x-axis. And y=145 is a vertical line that hits 145 on the x-axis. And y=0 is—you guessed it!—the y-axis.

It's all the other equations that get interesting… they're the ones that form slanted lines. One form of the equation of any straight line is ax +by=c. And any equation that graphs as a straight line (as opposed to a circle or other curve) is known as a Linear Equation.

Let's look at one of the ways to graph a straight line.

One method is through the use of the x- and y- intercepts. The x-intercept is the point where a line intercepts, of hits, the x-axis. And the y-intercept is the point where a line hits the y-axis.

To find each of the intercepts, set the OTHER variable equal to zero and solve. Then, once you have the 2 intercepts, plot them on a graph and connect with a straight line.

EXAMPLE: Find the intercepts of 2x + 5y =20.

ANSWER: Find the x-intercept by letting y=0. 2x+5(0)=20

2x = 2-

x=10.

The x-intercept is (10,0)

Find the y-intercept by letting x=0. 2(0) + 5y = 20

5y = 20

y=4

The y-intercept is (0,4)

To plot the line, plot (10,0) and (0,4), and connect them with a straight line.

BE CAREFUL!!! ANY time you plot the graph of an equation, your graph should extend to the ends of the x and y axes. The axes and your graph should all have arrows at both ends. And you should always label any graph with its equation.

PRACTICE PROBLEMS: Find the x and y-intercepts of each graph, and use them to plot the graph.

1. x+y = 5
2. 2x+y = 6
3. x – y = 4
4. x- 2y = 10
5. 2x + 3y = 12
6. 6x – y = 6
7. 5x-3y=30
8. 6x+2y = 18
9. 7x-y=14
10. 5x-2y=30
11. 11x=2y = 22
12. 12x- 3y = 24
13. 8x-3y = 12
14. 6x+2y = 24
15. 2x+6y= 10
16. 3x - 6y =18
17. -x +2y =10

PLOTTING A LINE USING A TABLE

Why did the two 4's skip lunch?

(They already 8!)

Another method of plotting a line is to set up a table and substitute in the x values of your choice. For example, if your equation is y=2x+3, this might be the table you set up:

x	Y=2x+3	y	(x,y)
0	2(0)+3	3	(0,3)
1	2(1)+3	5	-1.5
2	2(2)+3	7	-2.7

As you can see, I chose to use x values of 0, 1, and 2. I substituted them, one at a time, into the equation, and found the corresponding y values. From there, I came up with the ordered pair represented by that particular x and y value. Once I have 3 points, I can graph them on the Cartesian plane, connect them with a straight line, and label.

BE CAREFUL!!! (YET AGAIN!!!) ANY time you plot the graph of an equation, your graph should extend to the ends of the x and y axes. The axes and your graph should all have arrows at both ends. And you should always label any graph with its equation.

PRACTICE PROBLEMS: SET UP A TABLE AND USE IT TO PLOT THE GRAPH OF EACH EQUATION:

1. y = x-4
2. y= 2x+1
3. y = 8-x
4. y=4x-4
5. y=6x+2
6. y=10-2x
7. y = 3x-2
8. y = ½x -6
9. y = 4x
10. y= 3+x
11. x+y = 10
12. x-y=4
13. 2x+y = 9
14. x+y = 7
15. 3x+y = 12
16. x+y=0
17. 5x-10y=20
18. 2x+4y=8
19. 3x-y =9
20. 12-2x=y

66. SLOPE OF A LINE

A woman has seven daughters, and each daughter has a brother. How many children does the woman have all together?
(She has eight children!)

If you've ever been skiing, tubing or snowboarding, you know that there are lots of different trails on most mountains. They're rated—a green circle might be something a novice would enjoy, while a double diamond should only be attempted by someone with lots of experience.

What is it that differentiates one level from the other? It's not the length of the trail, since there are lots of nice long trails from the top of the mountain to its base.

No, its the SLOPE of the line that determines the level of difficulty.

Slope is a the ratio comparing how quickly the line goes up and how quickly the line moves to the left or right. It measures the steepness of the line. If the height increases a lot more quickly than the horizontal distance, then the line will be steep.

One way of expressing that is with the expression "Rise over Run." But in terms of actually using slope, most people prefer the formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

where m is the slope, and (x_1, y_1) and (x_2, y_2) are two of the points on the line.

BE CAREFUL!!!! When using the slope formula, be SURE to put the y's on top and the x's on the bottom!!!!

There are 2 special cases of slope. The first is for a horizontal line. When you plug the corresponding points into the formula for slope, you'll find that **THE SLOPE OF A HORIZONTAL LINE IS ALWAYS ZERO.**

The second special case is for a vertical line. When you plug the corresponding points into the formula for slope, you'll find that **VERTICAL LINES HAVE NO SLOPE.** That's because when you use the formula, you get a zero in the denominator, which means the slope is undefined.

On the other hand, a negative slope signifies that the line decreases, line this:

You'll need to keep those two ideas straight!!!

PRACTICE PROBLEMS:find the slope of the line connecting each pair of points:

1. (2, 2) and (5,5)
2. (2, 6) and (4, 12)
3. (-1, 4) and (1, -6)
4. (3, 7) and (7, 7)
5. (9, 0) and (12, 3)
6. (-4, 6) and (-10, 12)
7. (4, 7) and (4, 9)
8. (3, 7) and 7, 15
9. (6, 5) and (9, 6)
10. (10, 2) and (20, -4)
11. (3, 1) and (18, -6)
12. (14, 5) and (42, 9)
13. (0,0) and (4, -10)
14. (1,1) and (6,6,)
15. (7, 2) and (9, 4)
16. (3, 12) and (5, 20)
17. (0, 2) and (8, 4)
18. (9, 4) and (6, 3)
19. (0, 6) and (3, 0)
20. (5, 4) and (7, 8)
21. (2,4) and (6,2)
22. (5, 8) and (-8, -5)
23. (2,3) and (-2, -3)
24. (6, 3) and (8, 8

67. SLOPE-INTERCEPT FORM OF A LINE

Do you know a statistics joke?

(Probably, but it's mean!)

Any straight line that can be graphed has a unique equation. One of the most commonly used forms of the equation of a line is the slope-intercept form: y-mx+b.

In that equation, m is the slope and b is the y-intercept. As you may remember from a few pages back, the y-intercept is the point where the line intercepts (or crosses) the y-axis.

So a line with equation y=2x+6 has a slope of 2 and a y-intercept of 6.

PRACTICE PROBLEMS: Determine the slope and y-intercept of each graph:

1. y=2x+5
2. y=3x-4
3. y=-2x-8
4. y=5-3x
5. y=10-x
6. y=4x-3
7. y=12x+15
8. y= -x+6
9. y= ½x
10. y = 4x
11. x+y = 6
12. x-y=8
13. 2x+y =6
14. 3x-y=9
15. 5x+y = 10
16. 6x+3y = 12
17. 2x+3y = 9
18. 2x-5y=15
19. 3x-7y=14
20. 120x-20y=180

Write the equation of a line with:

21 slope = 4, y-intercept 3

22. slope 1/2, y-intercept -10

23. slope 1/8, y-intercept 5

24. slope -4/5, y-intercept -1

25. slope 6, y-intercept 0

26. slope 5/8, y-intercept 2

27. slope 6, y-intercept -6

28. slope 3/8, y-intercept 4

68. WRITING THE EQUATION OF A LINE

What kind of meals do math teachers eat?

(Square meals!)

Sometimes you know something about a line, and need to determine its equation. Depending on exactly what information you know, the process will change a bit. Here are the steps that need to be followed:

- Find the slope. If you're given the slope, great. If not, then use the formula for slope ($m = \frac{y_2 - y_1}{x_2 - x_1}$)

- Substitute that slope in for m in the equation y=mx+b.

- If you know the y-intercept, great. Substitute it in and you're done. But if you don't have the y-intercept, you'll need to find it. Choose a point on the line, and substitute in the x and y coordinates for the x and y respectively. Solve that equation for b.

- Your final equation should look like y=mx+b, with numbers in the "m" and "b" spots, and the x and y as variables.

PRACTICE PROBLEMS: FIND THE EQUATION OF THE LINE CONNECTING EACH PAIR OF POINTS:

1. (3, 4) and (5,6)
2. (2,5) and (4,9)
3. (-3,2) and(-5,4)
4. (16, 7) and (10, 4)
5. (5, 22) and (-2, -6)
6. (16, 2) and (20,3)
7. (5, 16) and (0,1)
8. (3, 7) and (3, 6)
9. (6, 1) and (9.2)
10. (5, 13) and (-2, -1)
11. (13, 4) and (5, -4)
12. (2, 6) and (-4, 6)
13. (7, 7) and (-8, -8)
14. (4, 14) and (8, 26)
15. (5, 0) and (10,1)
16. (20, 3)and (10, -7)
17. (-2, 6) and (4, 12)
18. (0,3) and (5, 23)
19. (0,0) and (4, 5)
20. (-1, 5) and (2, 7)

69. GRAPHING USING SLOPE AND Y-INTERCEPT

If you had 8 apples in one hand and 5 apples in the other, what would you have?

(Really big hands!)

You already know how to graph a line by plotting its x- and y-intercepts. An alternate method is through the use of the slope and y-intercept.

Here's what you do:

- Use y=mx+b to determine the slope and y-intercept.

- On your Cartesian plane, first plot the y-intercept on the y-axis. So, for example, if the y-intercept is 3, plot a point at (0,3)

- Next, use the slope. It needs to be a fraction, so if it's a whole number you can just put it over 1.

- From the y-intercept, count up the number of boxes in the numerator if your slope is positive, and down if it's negative.
- Move to the right the number of boxes in the denominator. Place another point there.

- Repeat the process, going up or down depending on the numerator and to the right depending on the denominator.

- Once you have 3 points, you can connect them with a straight edge. Be sure to put arrows on both ends of your graph and to label the graph with its equation
-

BE CAREFUL!!! There are a number of different approaches to the process we just followed. But if you use this process, you will ALWAYS move to the right. You may move up or down, but you will always move to the right, not the left.

BE CAREFUL!!! ANY time you plot the graph of an equation, your graph should extend to the ends of the x and y axes. The axes and your graph should all have arrows at both ends. And you should always label any graph with its equation.

PRACTICE PROBLEMS: DETERMINE THE SLOP AND Y-INTERCEPT OF EACH EQUATION, AND USE THEM TO SKETCH THE GRAPH.

1. $y=x+4$
2. $y=2x-3$
3. $y=5x-2$
4. $y=3x+6$
5. $y=-2x+7$
6. $y = -3x+4$
7. $y = -x+8$
8. $y=-3x- 5$
9. $y = ½ x -6$
10. $y = -\ \ x -5$
11. $y= 6-x$
12. $y = 10-3x$
13. $x+2y = 6$
14. $2x+ y =4$
15. $3x – y = -9$
16. $2x+3y = 6$
17. $3x – y = 12$
18. $6x - y= 2$
19. $–x + 2y = 8$
20. $3x + 4y = 12$
21. $2x+6y = 10$
22. $x - 2y = -4$
23. $x + y = 13$
24. $x - 4y = 8$
25. $4x + 2y = 12$
26. $2x + 5y = 15$
27. $3x - 4y = 12$
28. $2x + 2y = 2$
29. $y - x = 0$
30. $12x + 18 y = 36$

70. GRAPHIC SOLUTION TO A SYSTEM OF LINEAR EQUATIONS

What is a mathematician's favorite dessert?

(Pi!)

There are times when you have two (or more) equations, and you want to know which points those equations have in common. That's called a System of Equations, or Simultaneous Equations (because you want to solve them both at the same time—that's what "simultaneous" means.)

The systems of equations we'll be dealing with are Linear Equations, the same type we've been doing this year.

Right now we'll concentrate on solving them graphically.. .we'll branch out to algebraic solutions at some point down the road.

To solve a system of equations graphically, simply graph both equations on the same graph. You need to be VERY CAREFUL to be precise in your graphing, since being just a little off on just one line will throw off your answer.

If the two graphs intersect, that point of intersection is the solution for the system. Label that point with its coordinates.

BE CAREFUL!!! When you check a system of equations, plug the coordinates of the point of intersection into both original equations. It must check into BOTH original equations.

And I can't believe I'm typing this again, but years of experience have convinced me I should:

BE CAREFUL!!! (YET AGAIN!!!) ANY time you plot the graph of an equation, your graph should extend to the ends of the x and y axes. The axes and your graph should all have arrows at both ends. And you should always label any graph with its equation.

PRACTICE PROBLEMS: SOLVE GRAPHICALLY AND CHECK:

1. $y = 2x + 3$ and $x+y = 7$
2. $y=3x-1$ and $2x+3=y$
3. $y=1/2\ x -3$ and $x+y=9$
4. $2x +1 =y$ and $x+y=10$
5. $x - y = 4$ and $x+ y =6$
6. $3x + 2y = 12$ and $y=x+1$
7. $y=4x$ and $y=x+3$
8. $y=4-x$ and $y=x$
9. $3x-y =8$ and $y=x+2$
10. $y-2x = 3$ and $y-x =7$

71. LINEAR INEQUALITIES

What has eight legs and eight eyes?

(Eight pirates!)

There are many times when we speak of inequalities instead of equations. For example, the Speed Limit in many parts of the country is 55 mph. Does that mean that every car should be going 55 mph at every moment? Of course not, the intent behind the limit is that all cars should be going NO MORE than 55mph.

Likewise, you have to be at least 18 to vote…not “exactly 18” but of an age “greater than or equal to” 18.

We’ve solved a lot of inequalities in one variable. Now we’re going to graph inequalities in two variables.

The process is actually a lot like graphing equations:

- Solve the inequality for y.

- Temporarily replace the inequality with an equal sign. The line whose equation you create will be called the “border line.”

- Use one of the methods (chart, x-and y-intercepts, or slope-intercept) to graph the border line.

- If your original problem involved either or , then graph your border line with a solid line, as you have done previously. But if it involves either < or >, graph the border with a dotted (or “broken”) line.

- If your inequality involves < or , you’ll want to shade BELOW the border line. If it involves > or , you’ll be shading ABOVE the border line. When I say “shade” I mean you should be drawing a set of parallel lines—not random scribble, but something very neat. (THIS WILL MATTER down the road when we graph systems of inequalities. Please believe me… neatness counts here.)

- Label the graphed region, either with an “S” (for “solution”) or with the original inequality.

PRACTICE PROBLEMS: SKETCH THE GRAPH OF EACH INEQUALITY:

1. y> x-4
2. y < 2x
3. y<3x+6
4. y 2-x
5. y -3x+5
6. y<2x+8
7. y> ½x
8. y -x
9. y<3x+1
10. x+y >8
11. 2x+y 9
12. 10-2x>y
13. 15-3x<5y
14. y>-2x+1
15. 3x-2y <12
16. 5x-7y< 14

72. SYSTEMS OF LINEAR INEQUALITIES

TEACHER: "What's 2n plus 2n?"

STUDENT: "I don't know, it's 4n to me!"

You knew this one was coming… we've graphed single lines, systems of lines, and single inequalities. Now we're going to graph systems of inequalities.

Honestly, though, it's the same thing as graphing single inequalities. The ONLY difference is that this is the time, probably more than any other time this year, when neatness counts!!! If you're not careful with your shading, your paper will be a scribbled mess… and I can't give you credit for work I can't read.

So here's the process:

- Graph the first inequality. Shade the solution using parallel lines. Don't label anything yet, other than the mandatory x- and y- axes.
- Graph the second inequality. When it's time to shade it, choose a DIFFERENT direction from the direction you used with the first inequality. The idea is to have one region with one kind of shading, one with another, and the intersection to look almost like a plaid.

- That plaid area is labeled, either with "S" (for "solution") or with both original inequalities.

PRACTICE PROBLEMS: SOLVE EACH SYSTEM OF INEQUALITIES GRAPHICALLY.

1. $y < 3x$
$y > x-2$

2. $y \quad x - 6$
$y< x+10$

3. $y > x$
$y \quad 5$

4. $y > 3x - 1$
$y < 3x + 7$

5. $y< 5x+2$
$y > -x -3$

6. $y \quad 8$
$y \quad 4$

7. $y < 2x+5$
$y > 3x+1$

8. $y> \frac{1}{2} x-4$
$y < 3x+7$

9. $y \quad 4$
$x > -3$

10. $y > x+ 2$
$y < 5x+2$

11. $x + y <9$
$x - y > 2$

12. $2x+ y > 10$
$y <x + 3$

13. $3x - y > 6$
$x+ y \quad 9$

14. $y - x < 8$
$x+ 4y < 12$

77. ROOT OF A NUMBER

Why do plants hate math????
Because it gives them square roots

At this point, I'm sure you're familiar with the basic idea behind square roots: that the square root of a number is one of two equal factors of that number. So the square root of 25 is 5 because 5(5) = 25.

The symbol for square root is a radical symbol: √. The symbol for cube root is a radical with an index of 3.

What you may not realize is that it's also possible to take the cube root of a number. The cube root of 8, for example, is 2, because $2^3=8$. We can write that statement mathematically as $\sqrt[3]{8} = 2$. In that equation, the 3 is called the index, and the 8 is the radicand. It's also possible to indicate the 4^{th}, 5^{th}, or any other root, simply by changing the index.

There are some numbers whose square roots are integers. These numbers are referred to as **Perfect Squares**, and here are the first 13...you'll want to memorize them:

1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169.

I got that list by squaring 1, squaring 2, squaring 3, and so on up to 13.

I'll give you a shorter list of **Perfect Cubes:**

1, 8, 27, 64, 125, 216

Though of course both lists could go on forever.

We can, of course, work with the square roots of other numbers. The difference is that the square root of a perfect square is rational, while the square root of a non-perfect-square is an irrational number. is an example of an irrational number. To approximate the value of , find the two perfect squares on either side of : and . Our answer falls somewhere between those two values, which we know to be 2 and 3. Furthermore, since 5 is closer to 4 than to 9, we know that our

If we want both answers, the problem is written as .

OK, the question you're all thinking of: what if we want to take the square root of a negative number? The answer is that I can do it, but you can't, not yet. There is no real number whose square is, for example, -4. ($2^2 = 4$, $(-2)^2=4$; neither gives you -4) You'll learn about Imaginary numbers in Algebra II and Trig; until then, you can't take the square root of a negative number.

BUT.. you can take the cube root of a negative number. Think about if for a second... what number, if I cube it, will give me -8???

Not too hard, it's -2, because $(-2)(-2)(-2) = (-2)^3 = 8$.

PRACTICE PROBLEMS: EVALUATE EACH OF THE FOLLOWING

1. $\sqrt{25}$
2. $\sqrt{144}$
3. $\sqrt{169}$
4. $\sqrt{49}$
5. $\sqrt{64}$
6. $\sqrt{x^2}$
7. $\sqrt{1}$
8. $\sqrt{36x^2}$
9. $\sqrt[3]{8x^3}$
10. $(\sqrt{13})^2$
11. $(\sqrt{265})^2$

73. GRAPHING QUADRATIC EQUATIONS

Up until this point, all the equations we've graphed have been Linear Equations—first degree equations whose graph was a straight line. Today we're going to look at what happens when you graph a quadratic equation, one of the form $y=ax^2+bx+c$.

The graph that we'll be working with is called a Parabola. All the parabolas we'll be working with this year will open either up or down. (Sideways parabolas are the ones where the y is squared, not the x. You'll see them in Precalculus.)

Here's the process:

- Make sure your equation is in standard form.

- Identify the values of a, b, and c.

- If a >0, your parabola opens UP. If a<0, it opens DOWN.

- Find the equation of the Axis of Symmetry, using the formula $x=-b/2a$

- Set up a table of values. You'll want a column for the x value, one for the equation, and one for the y value.

- You'll need to plot at least 5 points. And—this is important—that value you got for the axis of symmetry needs to be the MIDDLE point. So put it in first, then build your x values up and down.

- Plugging the Axis of Symmetry into the equation gives you the coordinates of the **TURNING POINT.**

- Plotting those 5 points gives you the basis of your parabola. There will be some times when you'll need to add a point or two at either end.

- Also, sometimes it's helpful to plot the **ROOTS**—the points where the graph hits the x-axis. (Otherwise known as the **x-intercepts**.) You can find the roots by setting the equation equal to zero and factoring it, just as you did when we were solving Quadratic Equations.

- As always, remember to label your x and y axes, as well as the parabola.

BE CAREFUL!!! If the y values don't change direction at the turning point in the table of values, go back and check your work!!! The odds are good that you've made an error calculating the Axis of Symmetry.

BE CAREFUL!!! When you plot your parabola, keep in mind that the sides are CURVES, not straight lines. Probably the most iconic example I can give of a parabola is the world famous McDonalds golden arches.

BE CAREFUL!!!! Be particularly careful with your table of values when a is a negative value. Be sure to square your number first, and THEN multiply it by the a value. For some reason, this particular example of PEMDAS always causes a problem!!!

PRACTICE PROBLEMS: SKETCH THE GRAPH OF EACH OF THE FOLLOWING EQUATIONS:

1. $y = x^2+4x + 1$
2. $y= x^2 + 2x -3$
3. $y = x^2-6$
4. $y= x^2-2x+ 1$
5. $y=x^2 -6x +5$
6. $y= x^2 -4x +4$
7. $y = x^2 – 2x$
8. $y = x^2+4x$
9. $y = x^2 + 3x +2$
10. $y = x^2-5x +6$
11. $y = x^2 - x$
12. $y = 2x^2 +4x +3$
13. $y = 2x^2 - 6x + 4$
14. $y = -x^2$
15. $y = -x^2+2x +3$
16. $y = -x^2+6x+1$
17. $y = -2x^2$
18. $y= -3x^2 +6x + 2$
19. $y = -x^2+8x$
20. $y = -x^2+4x + 5$

74. ALGEBRAIC SOLUTION TO A SYSTEM OF EQUATIONS: ADDITION

The way that Google ranks the web pages on the internet is described with an algebraic expression. Once the information is collected a very large system of linear equations has to be solved in order to produce a database that can answer your query. This is a significant problem, how do you solve a system of linear equations with millions of equations and millions of unknowns? Brute force won't work, computers are just too slow. What is needed is a very sophisticated algebraic procedure

You're already aware that a system of equations can be solved graphically, by graphing both equations on the same graph and determining the point of intersection.

But sometimes an algebraic solution is called for. Graphing can sometimes take a while, or the solution to your system may be a fraction or decimal, one that's hard to pinpoint on a graph.

The basic rule is that you always need the same number of equations as you have variables. So a system with 2 variables will require 2 equations to solve; a system of 3 equations (which, sadly, you won't see until Junior year) requires 3.

Stop for a moment an consider these two equations that I'm going to add:

$2x + 3y = 36$
$\underline{1x - 3y = =18}$

What happens when you add those two equations? The y's cancel and you're able to solve for x, right??

WHY did those y's cancel each other out??

http://www.youtube.com/watch?v=itAOGRiYRLI

Because they had the same coefficients, but with opposite signs. And that's the key to this process:

- Rewrite your equations if necessary, so that the x's, y's, and constants fall into the following order: x's y's = constant.

- Decide which variable you're going to eliminate. If there's a variable where the coefficient in one equation is a multiple of the corresponding coefficient in the other, that will be easiest. Likewise, if there's a variable in either equation with a coefficient of 1, that will be easy to work with.

- Determine the Least Common Multiple of the coefficients of that particular variable.

-

- Multiply one or both equations by the factor needed to change the coefficient(s) to that LCM. Remember: you want the same coefficients, but with opposite signs.

- Add the equations. The variable you just worked on should cancel out, leaving you with one equation in one variable.

- Solve that equation for the variable it contains.

- Take that answer and substitute it into either of the original equations to find the 2nd value.

- If you are asked to check your answer, remember to check both answers into both original equations.

EXAMPLE: $2x + 3y = 3$
$3x + 4y = 5$

ANSWER: Since none of the coefficients is a multiple of the other, it makes no difference which variable I eliminate. I'll chose the x's, since their coefficients are slightly smaller than the y's.

The LCM of 2 and 3 is 6, so I'll want coefficients of 6 and -6 (it doesn't matter which equation has the -6). I'll multiply the first equation by 3 and the second by -2.

$3(2x + 3y = 3)$ $6x + 9y = 9$
$-2(3x + 4y = 5)$ $\underline{-6x - 8y = -10}$

$y = -1$

Once I have my y value, I can substitute it into any equation. I'll choose the very first equation: $2x + 3(-1) = 3$

$2x - 3 = 3$

$2x = 6$

$x = 3$

PRACTICE PROBLEMS: USE ADDITION TO SOLVE, THEN CHECK:

1. $6x+y = 15$, $-7x - 2y = -10$
2. $3x+2y = 4$, $8x - 3y = -6$
3. $5x + 4y = -1$, $-7x - 2y = -13$
4. $4x + \frac{1}{2} y = 50$, $-3x - 8y = -84$
5. $7x - \frac{1}{2} y = 30$, $6x + \frac{1}{2} y = 35$
6. $-3x - 3y = -24$, $4x + 5y = 42$
7. $4x + 4y = 44$, $5x - 3y = -57$
8. $10x - 3y = -32$, $-6x + 7y = 40$

75. ALGEBRAIC SOLUTION TO A SYSTEM OF EQUATIONS: SUBSTITUTION

Which month has 28 days?

All of them of course!

At this point, you know two methods of solving a system of linear equations: graphically and using addition. Today we're going to look at a third approach: substitution.

Like addition, substitution is an algebraic approach to solving a system of equations. And, as with addition, the point is to get from 2 equations in 2 variables down to 1 equation in one variable.

Here's the process:

- Decide which variable to eliminate. With substitution, this is best achieved when one of the coefficients in one of the equations is 1—that's the variable you want to solve for.

- Substitute that equation into the other, leaving you with a single equation containing a single variable.

- Solve for that variable.

- Take the answer you've gotten, and substitute it into either of the original equations to solve for the other variable.

- To check, substitute both values into both equations; the check MUST work for both equations.

EXAMPLE: $2x-y=-1$

$3x+2y=23$

ANSWER: Since the y in the 1st equation has a coefficient of 1, we'll choose to solve for that variable. Subtracting 2x from both sides, then dividing by -1 leaves us with y=2x+1.

We'll substitute that (2x+1) in for y in the other equation, giving us:

$3x+2(2x+1)=23$

$3x+4x+2=23$

$7x+2=23$

$7x=21$

$X=3.$

PRACTICE PROBLEMS: SOLVE USING SUBSTITUTION, AND CHECK:

1. $3x+3y=-18$
   $y=2x$

2. $x+4y=-3$
   $x=-12-7y$

3. $-3x+2y=-8$
   $y=-3-2x$

4. $-x-4y=-25$
   $x=18-3y$

5. $-\frac{1}{2}x-5y=5$
   $y=-4x-1$

6. $y=-\frac{1}{2}x+18$
   $y=-x+20$

7. $6x-2y=4$
   $Y=5x+3$

8. $x=-2y$
   $5x-y=44$

Which method do you use??

Without a graphing calculator, graphing is the most time consuming and the least accurate of the three methods, so use that only when the directions specify.

As to the other methods, technically it doesn't matter which method you use when. But as a general guideline, I would use Substitution when it was fairly easy to solve for one of the variables... when one of the variables in one of the equations has a coefficient of 1. The rest of the time I would opt for addition.

PRACTICE PROBLEMS: Solve using either method:

1. $-5x + 9y = -12$ $\quad$ $3x+2y = 22$
2. $y= -2.5x + 10$ $\quad$ $y= 0.5x+ 4$
3. $-3x+4y = -22$ $\quad$ $2x + 7y=5$
4. $-5x - y = -26$ $\quad$ $6x + 8y =4$
5. $3x+ 2y = 15$ $\quad$ $-4x - 5y = -13$
6. $y = x - 3$ $\quad$ $x + 4y = -1$
7. $-8x + 2y = -4$ $\quad$ $6x - 2y = 28$
8. $-15x - 20y = -50$ $\quad$ $5x+ 30y = 100$

IN THE MOOD FOR A CHALLENGE????

$2x - y + 3z = 37$

$x + y - 3z = -16$

$-3x + 5z = 24$

(Yes, that's all one problem. I'll give you a hint: use Addition with the first 2 equations to eliminate the y's... then see where it leads you.)

76. VERBAL PROBLEMS WITH 2 VARIABLES: BUSINESS PROBLEMS

What's the easiest way to double your money?
Put it in front of a mirror.

There are some problems where you need to find two quantities, and there's no direct relationship between them. In these instances, it's easiest to simply use two variables.

BE CAREFUL!!! Remember, you always need the same number of equations as you have variables. So if you have 2 variables, you're going to need 2 separate equations to solve, not merely 2 different phrasings of the same equation.

EXAMPLE: Six boxes of party hats and 5 boxes of noisemakers cost $142. At the same rates, 3 boxes of party hats and 2 boxes of noisemakers cost $64. Find the price of a box of party hats and the price of a box of noisemakers.

Answer: Let p= party hats

Let n= noisemakers

$6p+5n = 142$
$3p+2n= 64$
Since the first "p" coefficient is a multiple of the 2nd, I'll use addition. I'll multiply the 2nd equation by -2 to change the coefficient to -6.

$$\begin{array}{r} 6p + 5n = 142 \\ \underline{-6p - 4n =- 128} \\ 1n = 14 \end{array}$$

Now that I have the value for n, I can substitute it into either of the originals. I'll choose the 2nd equation: $3p + 2(14) =64$

$3p + 28 = 64$

$3p =36$

$P=12$.

The price of a box of party hats is $12 and the price of a box of noisemakers is $14.

Practice problems:

1. Katie spent $131 on shirts. Collared shirts cost $28 and T shirts cost $15. If she bought a total of 7 shirts, then how many of each type did she buy?
2. At Peter's Printing Company there are two kinds of printers: Model A can print 70 books per day and model B can print 55 books per day. The company owns a total of 14 printers, and this allows them to print 905 books per day. How many of each type of printer do they have?
3. At Tina's Toy Store, 3 trucks and 2 cars cost $56. Four trucks and 3 cars cost $78. Find the cost of a truck and the cost of a car.
4. At Beverly's Bake Shop, three pastries and two rolls cost $17. Six pastries and 6 rolls cost$30. Find the cost of a pastry and the cost of a roll.

78. SIMPLIFYING RADICALS

What 4 days of the week start with the letter 't'?

Tuesday, Thursday, today and tomorrow.

Much of the work we'll want to do with radicals requires that the radicals be simplified. Here's the process we use:

- Re-write the number as the product of two radicals, where the first factor is the largest possible perfect square. The other radical is the remaining factor.

- Take the square root of the perfect square; leave the other expression under the radical sign.

Let's talk about variables. Remember from the work you did earlier in the year that a variable raised to an EVEN power is a perfect square. To take its square root, divide the exponent by 2. (In the case of cube roots, an exponent that's a multiple of 3 is a perfect cube; divide it by 3 to find the cube root.)

EXAMPLE: SIMPLIFY $\sqrt{75}$

ANSWER: The largest perfect square that's a factor of 75 is 25. So we rewrite our problem as: $\sqrt{25}\cdot\sqrt{3}$. Now simplify the perfect square, and our answer is $5\sqrt{3}$

PRACTICE PROBLEMS: SIMPLIFY EACH RADICAL:

1. $\sqrt{8}$
2. $\sqrt{12}$
3. $\sqrt{20}$
4. $\sqrt{27}$
5. $\sqrt{50}$
6. $\sqrt{72}$
7. $\sqrt{28}$
8. $\sqrt{54}$
9. $\sqrt{200}$
10. $\sqrt{75}$
11. $\sqrt{44}$
12. $\sqrt{108}$
13. $\sqrt{242}$
14. $\sqrt{128}$
15. $\sqrt{24x^4}$
16. $\sqrt{32x^5}$
17. $3\sqrt{98x^3}$
18. $2\sqrt{162x^5}$
19. $\sqrt{150x^5}$
20. $\sqrt{48x^7}$
21. $3\sqrt{18}$
22. $\sqrt{54}$
23. $5\sqrt{175}$
24. $2\sqrt{24}$

79. ADDITION AND SUBTRACTION OF RADICALS

Teacher: Your behavior reminds me of square root of 2?

Student: Why?

Teacher: Because its' completely irrational.

Adding and subtracting radicals is a lot like adding and subtracting variables: you can only add or subtract like terms. The way to get like terms is to simply your radicals. Once your radicals have been simplified completely, you can add radical 2's to radical 2's, radical 3's to radical 3's and so on.

EXAMPLE: ADD: $\sqrt{2} + \sqrt{8}$

ANSWER: Our first step is to simplify each of the radicals:

$\sqrt{2} + \sqrt{4}(\sqrt{2})$

$1\sqrt{2} + 2\sqrt{2}$

$3\sqrt{2}$

BE CAREFUL!!! It's important to realize that your radicals can't just be added from the beginning; they MUST be simplified to the point where they're like terms before you add or subtract!

PRACTICE PROBLEMS:

1. $\sqrt{5} + \sqrt{20}$
2. $\sqrt{8} - \sqrt{2}$
3. $\sqrt{150} - \sqrt{24}$
4. $\sqrt{20} + \sqrt{45})$
5. $\sqrt{12} - \sqrt{27}$
6. $\sqrt{75} + \sqrt{27}$
7. $\sqrt{72} - \sqrt{108}$
8. $\sqrt{200} + \sqrt{300}$
9. $\sqrt{45} - \sqrt{125}$
10. $\sqrt{48} - \sqrt{27}$
11. $\sqrt{162} - \sqrt{8}$
12. $\sqrt{18} + \sqrt{242}$
13. $\sqrt{54} - \sqrt{24}$
14. $\sqrt{27} + \sqrt{75}$
15. $\sqrt{80} + \sqrt{125}$
16. $\sqrt{8} + \sqrt{32} - \sqrt{50}$
17. $\sqrt{500} - \sqrt{45}$
18. $\sqrt{20} - \sqrt{80}$
19. $\sqrt{12} + \sqrt{175}$
20. $\sqrt{2x} + \sqrt{72X}$
21. $\sqrt{300} + 4\sqrt{147}$
22. $2\sqrt{98} - \sqrt{128}$
23. $3\sqrt{75} + 2\sqrt{108}$
24. $3\sqrt{128X} + \sqrt{200X}$
25. $\sqrt{363} - 2\sqrt{48}$
26. $\sqrt{50X} + \sqrt{98X}$
27. $5\sqrt{20X} - 2\sqrt{45X}$
28. $10\sqrt{(300X^2)} + 3\sqrt{(75X^2)}$
29. $10\sqrt{300X} + 3\sqrt{75X}$
30. $6\sqrt{162x} + 4\sqrt{242x} - \sqrt{18x}$

80. MULTIPLICATION OF RADICALS

Teacher: Let's find the square root of 1 million.

Student: Don't you think that's a bit too radical?

Multiplication of radicals is pretty much as you would guess it to be:

- First, multiply any coefficients
- Next, multiply all radicands.
- Finally, simplify the product.

EXAMPLE: FIND THE PRODUCT (2)(6)

ANSWER: When we multiply the coefficients, we get 2(6) or 12. When we multiply the radicands, we get . So far, our answer is 12 , which of course needs to be simplified. Since 90 = 9(10), we get 12(3 , or 36

A NICE LITTLE SHORTCUT: When you multiply a square root by itself, (or square it) you get the expression that was under the radical sign. So , for example, $\sqrt{(6)}^2 = 6$.

Practice Problems

81. DIVIDING RADICALS

If two's company and three's a crowd, what are four and five? 9

Division of radicals starts off pretty predictably: divide the coefficients, then divide the radicands and simplify… easy peasy, right???

But wait!!! Just when you thought you had all this algebra stuff figured out, there's a strange little rule you need to follow: You can NEVER, (as in "never ever") leave a radical in the denominator of a fraction.(I know of no logical reason why it's such an awful affront to humanity, I just know that it's a rule we follow—kind of why we stop at a red light and go on a green—why not orange and purple? Or blue and gold???)

Anyway, the process of getting that radical out of the denominator is called Rationalizing the denominator. And here's how you do it:

- If the denominator is a monomial, just multiply top and bottom by the radical that's in the bottom, then simplify both the numerator and denominator. The radical will be out of the denominator.

- If the denominator is a binomial, things get kind of interesting. Multiply the top and bottom of the fraction by the Conjugate of the bottom—that's the same expression, but with an opposite sign in the middle. (So, for example, the conjugate of $(2+\sqrt{3})$ is $(2-\sqrt{3})$. You may have to FOIL the top, though the bottom multiplies to be the difference of perfect squares.

EXAMPLE: DIVIDE $4 \div \sqrt{2}$

ANSWER: Straight division gives us $2/\sqrt{2}$. But now we need to rationalize that denominator. Multiply top and bottom by $\sqrt{2}$. We get $2\sqrt{2}/2$ OR $\sqrt{2}$.

PRACTICE PROBLEMS: EXPRESS IN SIMPLEST FORM:

1. $12\sqrt{20} / 3\sqrt{5}$
2. $6\sqrt{6} / \sqrt{2}$
3. $20\sqrt{18} / 4\sqrt{6}$
4. $36\sqrt{40} / 2\sqrt{10}$
5. $72\sqrt{150} / 4\sqrt{50}$
6. $100\sqrt{20} / 2\sqrt{5}$
7. $1/\sqrt{2}$
8. $6/\sqrt{3}$
9. $12/\sqrt{3}$
10. $9\sqrt{60} / 3\sqrt{5}$
11. $40\sqrt{140} / 8\sqrt{7}$
12. $72\sqrt{32} / 16\sqrt{4}$
13. $18/\sqrt{6}$
14. $27/\sqrt{9}$
15. $54/\sqrt{6}$
16. $30/\sqrt{6}$
17. $50/\sqrt{10}$
18. $1/(2+\sqrt{2})$
19. $12/(6-\sqrt{2})$
20. $16/(\sqrt{2}+1)$
21. $8/(2+\sqrt{2})$
22. $6/(2-\sqrt{3})$
23. $18/(3-\sqrt{3})$
24. $96/(5-\sqrt{2})$
25. $1/(\sqrt{3}-1)$
26. $2/(4-\sqrt{2})$
27. $4/(\sqrt{6}+2)$
28. $90/(\sqrt{1}+2)$
29. $72/(6-\sqrt{3})$
30. $4/(\sqrt{6}-2)$

82. EQUATIONS WITH RADICALS

What occurs once in every minute, twice in every moment and yet never in a thousand years?

The letter m.

When equations contain a variable under a radical sign, the process is pretty predictable—you need to isolate that term. Once it's on one side of the equation all by itself, you square both sides—that will sometimes require FOIL or other Polynomial Multiplication on the other side. (And remember, squaring a square root gives you whatever was under the radical.)

BUT… like a number of other types of equations we've spoken about this year, these MUST be checked for extraneous roots.

EXAMPLE: Solve: $3 + \sqrt{(x+4)} = 12$

ANSWER: $\sqrt{(x+4)} = 9$

$(\sqrt{(x+4)})^2 = 9^2$

$x+4 = 81$

$x = 77$

CHECK: $3 + \sqrt{(x+4)} = 12$

$3+\sqrt{(77+4)} = 12$

$3+9 = 12$

$12 = 12$

$\{77\}$

PRACTICE PROBLEMS: SOLVE AND CHECK:

1. √(x+1) =6
2. 2+ √ (x+3) =5
3. √(x-5) + 6 =10
4. √2x -4 =8
5. √ (x-5) +5 =6
6. √(2x+1) +3 = 10
7. √ (2x+5 -7 =9
8. √3x = 12
9. √ x-2 + 4 =1
10. √ 3x + 6 = 9
11. √4x - 2 = 14
12. 2x+1 = 12
13. √x /2 = 5
14. 2√(x-3) =6
15. 4v(2x+1) =18
16. √(x-5) /2 =7
17. 3√2x =18
18. √(3x+3) =9
19. -2√ (x+6) =-4
20. √(x-2) = x - 4

83. PYTHAGOREAN THEOREM

If there are 4 apples and you take away 3, how many do you have?

You took 3 apples so obviously you have 3.

Once upon a time, in a land far away, there was no internet. There wasn't even TV. No Ipads or Ipods , no Facebook or Kik or Instagram or Twitter. There was nothing to do but sit around all day and think about math.

One of those thinkers was a guy named Pythagoreas. He did a lot of thinking about math, and contributed a lot. There was even a secret society of his believers: http://fclass.vaniercollege.qc.ca/web/F0000EB84/people/pythag.htm

But he's best known for the world famous Pythagorean Theorem… which he may or may not have come up with. http://ualr.edu/lasmoller/pythag.html

As you're probably aware, here's what it states: The sum of the squares of the lengths of the legs of a right triangle equals the square of the length of the hypotenuse.

Or, in the terms you're probably more used to: $a^2 + b^2 = c^2$ where a and b are the lengths of the legs of a right triangle and c is the length of the hypotenuse.

EXAMPLE: The lengths of the legs of a right triangle are 5 and 12. Find the length of the hypotenuse

ANSWER: Since we were given the legs, we have a and b—it doesn't matter which leg is substituted for which of those two variables.

$5^2 + 12^2 = c^2$

$25 + 144 = c^2$

$169 = c^2$

$13 = c$

PRACTICE PROBLEMS: Use Pythagorean Theorem to find the third side of the triangle. ("c"= the hypotenuse, "a" and "b" are legs.) Express any radicals in simplest form.

1. a=3, b = 6
2. a=6, c = 10
3. a = 5, c = 13
4. b = 8, c = 17
5. a = 14, b = 28
6. a = 2, b = 6
7. a = 5, c = 9
8. a = 4, c = 8
9. a = 1, b=2
10. a = 6, b = 10
11. a = 12, b = 14
12. a = 4, c = 12
13. a = 5, c = 10
14. a = 7, c = 9
15. a = 4, c = 12
16. The sides of a right triangle are consecutive integers. Find them.
17. The sides of a right triangle are consecutive even integers. Find them.
18. One leg of a right triangle is 2 more than twice another. The length of the hypotenuse is one more than the longer leg. Find all three sides.
19. The legs of a right triangle are (x-1), (x+2) and (x+5). Find them.
20. The legs of a right triangle are (x-1), (x+6) and (2x-1). Find them.
21. Find the diagonal of a square whose sides are 4.
22. Find the diagonal of a rectangle whose sides are 4 and 6.
23. The diagonal of a square is 12. Find the length of a side.
24. The length of a rectangle is twice its width. Its diagonal is 12. Find the lengths of its sides.
25. The sides of a right triangle are (x-1), (x+2) and (x+5). Find them.
26. The legs of a right triangle are (x-1) and (x+6) and the hypotenuse is (2x-1). Find all 3 sides.

84. Why Learn Algebra?

by Jason Gibson

"Why study algebra?" If you're a parent, it's a question that you will no doubt hear as your children study the subject. If you're a student, it is a very natural question to ask, "What's the point of learning algebra in the first place?"

After all, all of the math leading up to algebra that we learned growing up such as addition, multiplication, decimals, fractions, and the like, seem to have a concrete meaning. These concepts all deal with numbers in some way or another and because of this we can wrap our brains more easily around the concepts. After all, I can pick up six pencils and give two to a friend and by using math I can figure out how many pencils I am left holding in my hand. We can all imagine situations where basic math serves us well - calculating your change in the grocery store for instance.

In short, basic math deals with numbers. Since we are all taught how to count at a young age the concepts of basic math, even though challenging at first, seem to have a practical value - even to children.

Enter Algebra. Suddenly, we are asked to deal not only with our comfortable numbers but with letters. And it doesn't stop with this. You start seeing parenthesis and exponents, and a whole potpourri of other symbols that seem to make no sense at all. This single fact more than any other turns many people off to learning algebra. At the very beginning you are asked to learn certain rules on how to calculate things in algebra. You must learn which steps are legal to do before others, and if you do them in the reverse order you get the wrong answer!

This leads to frustration. With frustration, despair follows in short order. And so the thoughts begin:

"Why do I need to learn this?"

"When would I ever use Algebra in real life?"

What you have to remember, though, is that basic math is riddled with special rules and symbols as well. For example, the symbols "+" and "=" were at one time foreign to us all. In addition the concept of adding fractions, as a single example, is filled with special rules that we must learn. When adding 1/3 to 1/3, for example, you keep the common denominator and add the numerators, so that $1/3 + 1/3 = 2/3$. The point here is that when you begin to learn algebra it may seem overwhelming with the rules that you must learn, but this is no different from the multitude of rules that you had to learn that dealt with basic math such as addition and subtraction.

But this does not answer the question of "Why should I learn Algebra?" This is a difficult question, but the simplest answer is that Algebra is the beginning of a journey that gives you the skills to solve more complex problems.

What types of problems can you solve using only the skills you learned in Algebra? I invite you to take a journey with me back to your childhood. We've all been to the playground and had a great time on the see-saw, the merry-go-round, and the slide. At one time all of us were completely fascinated with these trips to the playground, but Algebra can help you understand them. The physics of all of these playground toys can be completely understood using only Algebra. No Calculus required. For example, if you knew the weight of a person at the top of the slide and you knew the height of the slide you could roughly calculate how fast you would be traveling as you exited the bottom of the slide.

On the see-saw, let's say that a person was sitting at one end and you knew that person's weight. You'd like to sit on the other side of the see-saw, but not at the very end - you'd like to sit opposite your partner in the middle between the seat and the pivot point. Using algebra, you could calculate how heavy you'd have to be to exactly balance the see-saw.

Moving away from playground equipment, as children we were all fascinated with the magical way that magnets attract each other. Using algebra, you could calculate how much force a given magnet would pull on another magnet.

There are examples all around us of things in the everyday world that you could fully understand using only the tools in algebra. If you drop a rock off of the roof of a house, how long would it take to hit the ground? If you dropped a second rock 100 times as heavy off of the roof of the same house, how long would it take to hit the ground? If you somehow brought a bulldozer up to the roof of the house and dropped it, how long would it take for the bulldozer to hit the ground? The answer in all three cases it takes the same amount of time to hit the ground! The time of free-fall depends only on the Earth's gravitational field (which is the same for us all) and the height of the roof you drop from. Even though the bulldozer is "heavier" than the rocks, they all fall at the same rate to the ground.

Most people would assume that learning about more "advanced" topics such as rocket propulsion and Einstein's theory of Relativity would require much more advanced math than Algebra. It is true that more advanced math is necessary to understand every facet of these and other advanced topics. However, many of the fundamental principles can be understood using only the tools in algebra. For example, the equations that describe how a spacecraft orbits the Earth only involve algebra.

Moreover, many of the central topics in Einstein's theory of special relativity can be understood only using algebra. For example, it turns out if you are traveling on a spaceship near the speed of light time actually slows down for you relative to your friends back on Earth. In other words, if you were to fly in a spaceship near the speed of light for some time and then you returned to Earth, you would find that you had aged very little while your friends on Earth have aged a great deal! Albert Einstein coined this phenomenon "time dilation" and it can easily be calculated using only Algebra. This effect is not a theoretical effect - it has actually been measured many times. In fact, the GPS system of satellites in the sky that the military and police forces depend on must take into account the effects of time dilation or else the system would not work at all! Because the satellites are moving in orbit around the Earth at speeds much smaller than the speed of light, the time dilation involved is very small - but it must be accounted for or the system would not function.

Now, you might be thinking, "I never learned how to calculate things such as this in my algebra class!" This is in fact true. All of the applications we have been talking about here are known as the study of Physics. If you had to boil the word Physics down to one sentence it would be: "Physics is all about studying the world around us using math as a tool."

Simply put all the math that you ever learn is really a tool for understanding the world around us. And believe me, we have only begun to scratch the surface of understanding how the world works. Algebra is a stepping stone to learning about this wonderful universe that we live in. With it you have the tools to understand a great many things and you also have the skills needed to continue on and learn Trigonometry and Calculus which are essential for exploring other types of problems and phenomena around us.

So, try not to think of Algebra as a boring list of rules and procedures to memorize. Consider algebra as a gateway to exploring the world around us all."

And, just in case that didn't convince you, http://www.csub.edu/~lwildman/comappAlge1std.htm

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